Dos depositos

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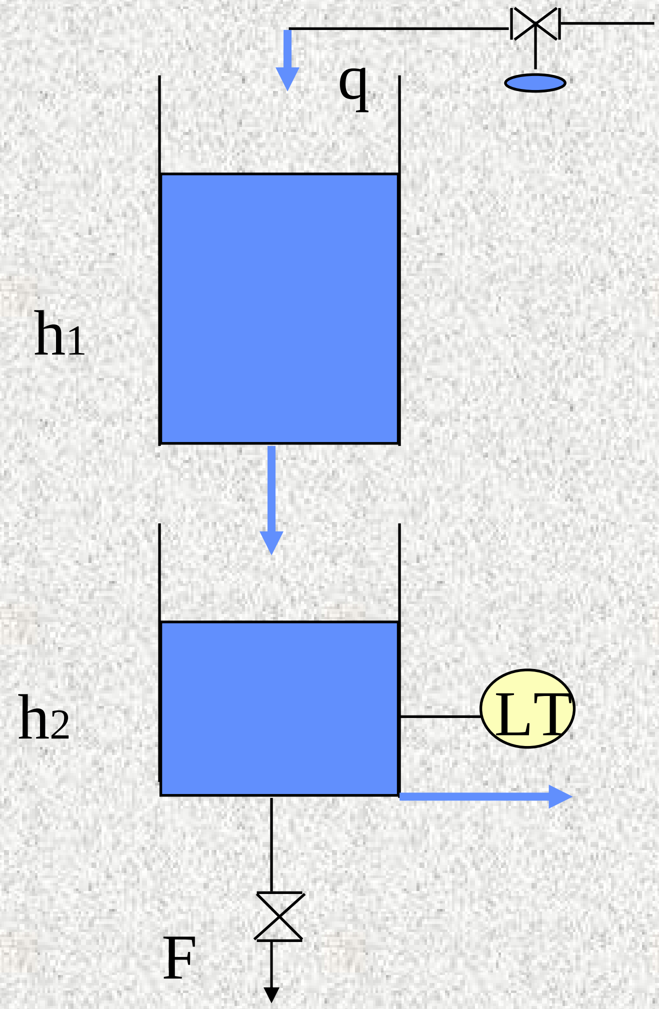
Dos depósitos



Modelo dinámico:

Balances de masa en ambos depósitos

h_i nivel del líquido

A_i sección del depósito

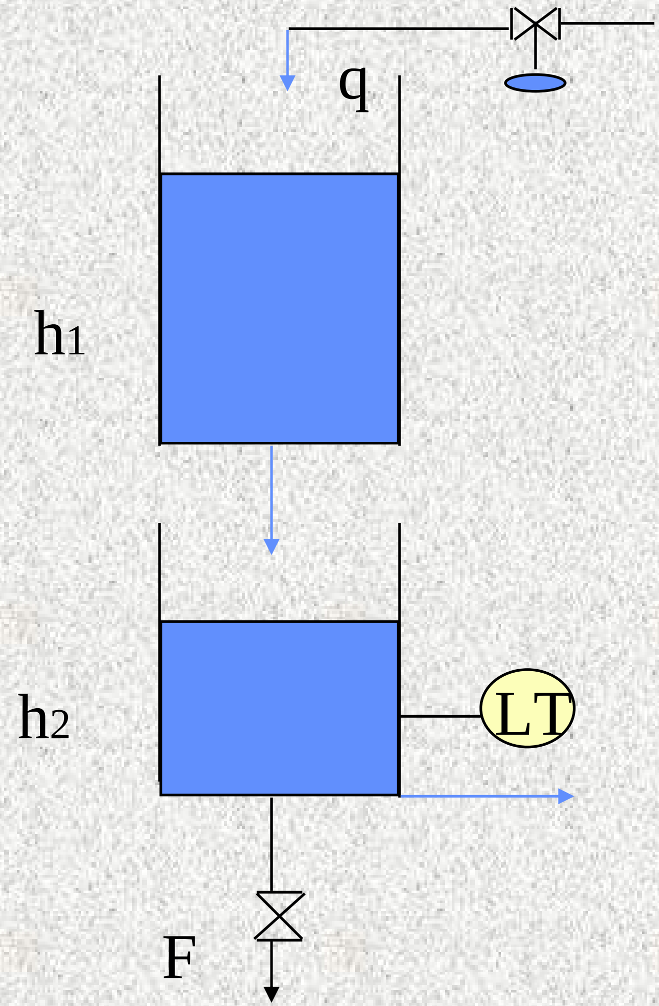
q caudal de entrada

F caudal fijo (perturbación)

$$A_1 \frac{dh_1}{dt} = q - k_1\sqrt{h_1}$$

$$A_2 \frac{dh_2}{dt} = k_1\sqrt{h_1} - k_2\sqrt{h_2} - F$$

Estimación de parámetros



Medidas en estado estacionario
Documentación o medidas de A_i

$$0 = q - k_1\sqrt{h_1}$$

$$0 = k_1\sqrt{h_1} - k_2\sqrt{h_2} - F$$

Medidas en estado estacionario:

q=17.8 l/m h_2= 40 dm F=2 l/m

q=25 l/m h_2= 84.5 dm F=2 l/m
h_1=100 dm

Estimación de parámetros

$$0 = q - k_1\sqrt{h_1}$$

$$0 = k_1\sqrt{h_1} - k_2\sqrt{h_2} - F$$

Medidas:

q=17.8 l/m h_2= 40 dm F=2 l/m

q=25 l/m h_2= 84.5 dm F=2 l/m
h_1=100 dm

A_1= 0.2 dm^2 A_2=0.2 dm^2

$$0 = 17.8 - k_1\sqrt{h_{10}}$$

$$0 = 17.8 - k_2\sqrt{40} - 2$$

k_2 = 15.8/6.324=2.5

$$0 = 25 - k_1\sqrt{100}$$

$$k_1 = 2.5$$

$$0 = 17.8 - k_1\sqrt{h_{10}}$$

$$h_{10} = \left(\frac{17.8}{2.5}\right)^2 = 50.69$$

Modelo linealizado (1)

$$A_1 \frac{dh_1}{dt} = q - k_1 \sqrt{h_1}$$

$$A_2 \frac{dh_2}{dt} = k_1 \sqrt{h_1} - k_2 \sqrt{h_2} - F$$

Punto de operación:

q=17.8 l/m h_{20}= 40 dm

F=2 l/m h_{10}= 50.69 dm

A_1=0.2 dm^2 A_2=0.2 dm^2

$$A_1 \frac{d\Delta h_1}{dt} = \Delta q - \frac{k_1}{2\sqrt{h_{10}}} \Delta h_1$$

$$A_2 \frac{d\Delta h_2}{dt} = \frac{k_1}{2\sqrt{h_{10}}} \Delta h_1 - \frac{k_2}{2\sqrt{h_{20}}} \Delta h_2 - \Delta F$$

Modelo en variables de estado

$$\frac{d\,\Delta h_1}{d\,t} = \boxed{\frac{1}{A_1}}\Delta q - \boxed{\frac{k_1}{2A_1\sqrt{h_{10}}}}\Delta h_1$$

$$\frac{d\,\Delta h_2}{d\,t} = \boxed{\frac{k_1}{2A_2\sqrt{h_{10}}}}\Delta h_1 - \boxed{\frac{k_2}{2A_2\sqrt{h_{20}}}}\Delta h_2 - \boxed{\frac{1}{A_2}}\Delta F$$

$$\frac{d\Delta h_1}{dt} = b_1\Delta q + a_{11}\Delta h_1 = 5\Delta q - 0.878\Delta h_1$$

$$\frac{d\Delta h_2}{dt} = a_{21}\Delta h_1 + a_{22}\Delta h_2 + d_2\Delta F =$$

$$= 0.878\Delta h_1 - 0.99\Delta h_2 - 5\Delta F$$

Modelo en variables de estado

$$\begin{bmatrix} \Delta\dot{h}_1 \\ \Delta\dot{h}_2 \end{bmatrix} = \begin{pmatrix} a_{11} & 0 \\ a_{21} & a_{22} \end{pmatrix} \begin{bmatrix} \Delta h_1 \\ \Delta h_2 \end{bmatrix} + \begin{pmatrix} b_1 & 0 \\ 0 & d_2 \end{pmatrix} \begin{bmatrix} \Delta q \\ \Delta F \end{bmatrix}$$

$$\Delta h_2 = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{bmatrix} \Delta h_1 \\ \Delta h_2 \end{bmatrix} + \begin{pmatrix} 0 & 0 \end{pmatrix} \begin{bmatrix} \Delta q \\ \Delta F \end{bmatrix}$$

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

$$\begin{bmatrix} \Delta\dot{h}_1 \\ \Delta\dot{h}_2 \end{bmatrix} = \begin{pmatrix} -0.878 & 0 \\ 0.878 & -0.99 \end{pmatrix} \begin{bmatrix} \Delta h_1 \\ \Delta h_2 \end{bmatrix} + \begin{pmatrix} 5 & 0 \\ 0 & -5 \end{pmatrix} \begin{bmatrix} \Delta q \\ \Delta F \end{bmatrix}$$

$$\Delta h_2 = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{bmatrix} \Delta h_1 \\ \Delta h_2 \end{bmatrix} + \begin{pmatrix} 0 & 0 \end{pmatrix} \begin{bmatrix} \Delta q \\ \Delta F \end{bmatrix}$$

Modelo tipo función de transferencia

$$\frac{d\,\Delta h_1}{d\,t} = b_1 \Delta q + a_{11} \Delta h_1$$

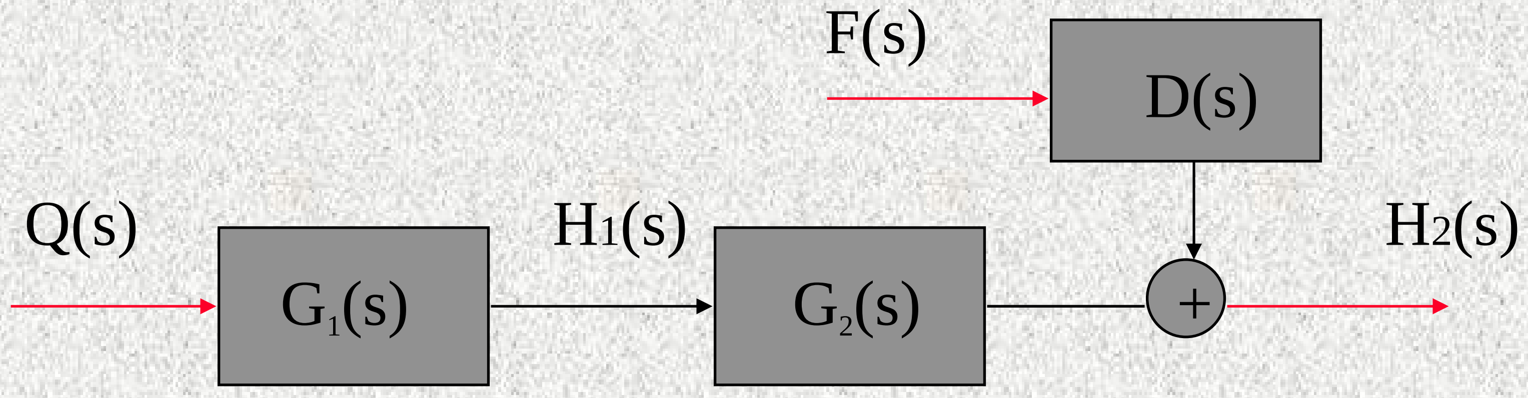
$$\frac{d\,\Delta h_2}{d\,t} = a_{21} \Delta h_1 + a_{22} \Delta h_2 + d_2 \Delta F$$

$$sH_1(s) = b_1 Q(s) + a_{11} H_1(s)$$

$$H_1(s) = \frac{b_1}{s - a_{11}} Q(s) = G_1(s) Q(s)$$

$$H_2(s) = \frac{a_{21}}{s - a_{22}} H_1(s) + \frac{d_2}{s - a_{22}} F(s) = G_2(s) H_1(s) + D(s) F(s)$$

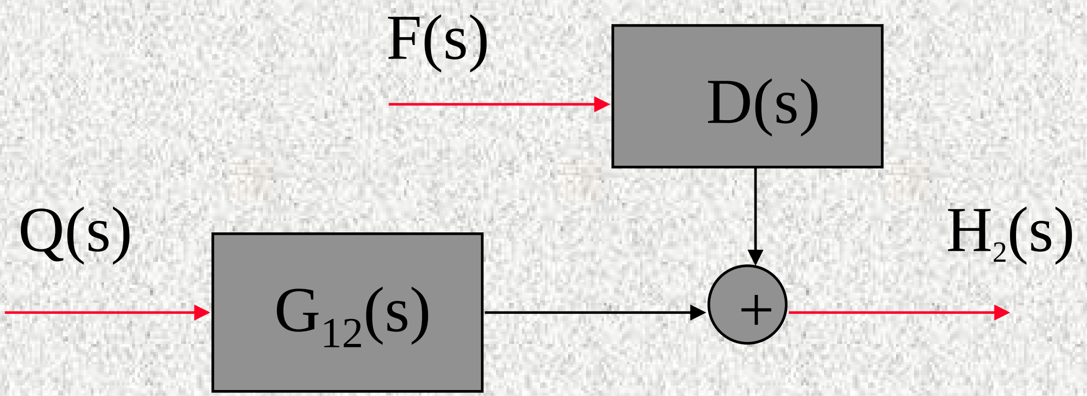
Diagrama de bloques(1)



$$H_1(s) = G_1(s)Q(s)$$

$$H_2(s) = G_2(s)H_1(s) + D(s)F(s) = G_2(s)G_1(s)Q(s) + D(s)F(s) =$$
$$= G_{12}(s)Q(s) + D(s)F(s)$$

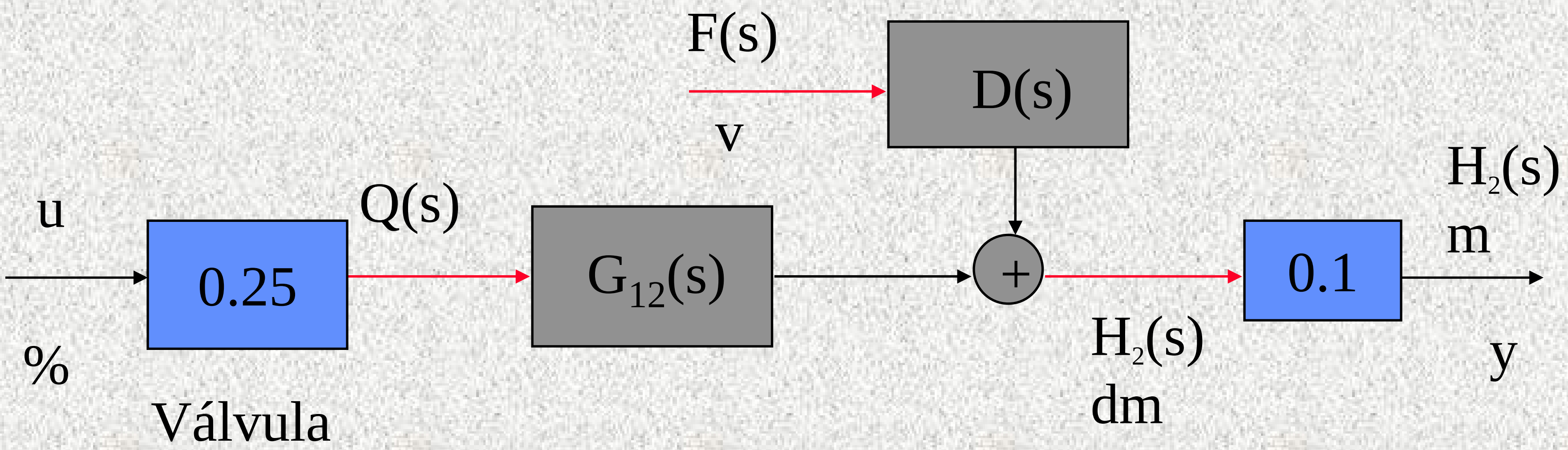
Diagrama de bloques(2)



$$H_2(s) = G_{12}(s)Q(s) + D(s)F(s) = \frac{4.39}{(s+0.878)(s+0.99)}Q(s) - \frac{5}{s+0.99}F(s) =$$

$$= \frac{5.05}{(1.01s+1)(1.14s+1)}Q(s) - \frac{5.05}{1.01s+1}F(s)$$

Unidades / Válvula



$$H_2(s) = G(s)U(s) + D(s)F(s) = \frac{0.126}{(1.01s+1)(1.14s+1)}U(s) - \frac{0.505}{1.01s+1}F(s) =$$

$$= \frac{0.11}{s^2 + 1.8673s + 0.8685}U(s) - \frac{0.505}{1.01s+1}F(s)$$

Función de transferencia. Cálculo alternativo

(1) $$A_2 \frac{d\Delta h_2}{dt} = \frac{k_1}{2\sqrt{h_{10}}}\Delta h_1 - \frac{k_2}{2\sqrt{h_{20}}}\Delta h_2 - \Delta F$$

(2) $$A_1 \frac{d\Delta h_1}{dt} = \Delta q - \frac{k_1}{2\sqrt{h_{10}}}\Delta h_1$$

Derivando (1)

$$A_2 \frac{d^2\Delta h_2}{dt^2} = \frac{k_1}{2\sqrt{h_{10}}}\frac{d\Delta h_1}{dt} - \frac{k_2}{2\sqrt{h_{20}}}\frac{d\Delta h_2}{dt} - \frac{d\Delta F}{dt}$$

(2)

$$A_2 \frac{d^2\Delta h_2}{dt^2} = \frac{k_1}{2\sqrt{h_{10}}A_1}\left[\Delta q - \frac{k_1}{2\sqrt{h_{10}}}\Delta h_1\right] - \frac{k_2}{2\sqrt{h_{20}}}\frac{d\Delta h_2}{dt} - \frac{d\Delta F}{dt}$$

(1)

$$A_2 \frac{d^2\Delta h_2}{dt^2} = \frac{k_1}{2\sqrt{h_{10}}A_1}\left[\Delta q - A_2\frac{d\Delta h_2}{dt} - \frac{k_2}{2\sqrt{h_{20}}}\Delta h_2 - \Delta F\right] - \frac{k_2}{2\sqrt{h_{20}}}\frac{d\Delta h_2}{dt} - \frac{d\Delta F}{dt}$$

Función de transferencia. Cálculo alternativo

$$\frac{d^2\Delta h_2}{dt^2}+\left[\frac{k_2}{2A_2\sqrt{h_{20}}}+\frac{k_1}{2\sqrt{h_{10}}A_1}\right]\frac{d\Delta h_2}{dt}+\frac{k_1k_2}{4A_1A_2\sqrt{h_{20}h_{10}}}\Delta h_2=$$

$$=\frac{k_1}{2A_1A_2\sqrt{h_{10}}}\Delta q-\frac{k_1}{2A_1A_2\sqrt{h_{10}}}\Delta F-\frac{1}{A_2}\frac{d\Delta F}{dt}$$

Tomando transformadas:

$$\left[s^2+\left(\frac{k_2}{2A_2\sqrt{h_{20}}}+\frac{k_1}{2\sqrt{h_{10}}A_1}\right)s+\frac{k_1k_2}{4A_1A_2\sqrt{h_{20}h_{10}}}\right]H_2(s)=$$

$$=\frac{k_1}{2A_1A_2\sqrt{h_{10}}}Q(s)-\frac{1}{A_2}\left[\frac{k_1}{2A_1\sqrt{h_{10}}}+s\right]F(s)$$

La raiz s+b es comun en F

$$H_2(s)=\frac{b/A_2}{s^2+(a+b)s+ab}Q(s)-\frac{1/A_2}{s+a}F(s)$$

$$a=\frac{k_2}{2A_2\sqrt{h_{20}}} \qquad b=\frac{k_1}{2A_1\sqrt{h_{10}}}$$

Función de transferencia. Cálculo alternativo

$$Y(s)=\frac{K\omega_n^2}{s^2+2\delta\omega_n s+\omega_n^2}U(s)+\frac{K'}{\tau s+1}V(s) \qquad a=\frac{k_2}{2A_2\sqrt{h_{20}}} \qquad b=\frac{k_1}{2A_1\sqrt{h_{10}}}$$

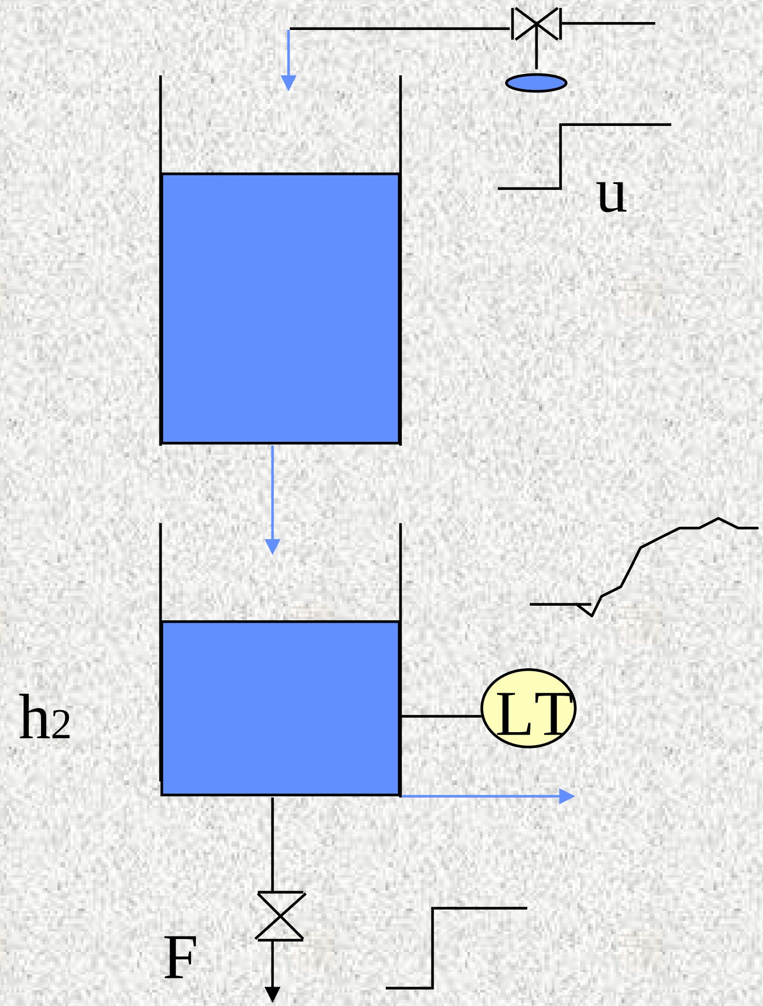
$$H_2(s)=\frac{b/A_2}{s^2+(a+b)s+ab}Q(s)-\frac{1/A_2}{s+a}F(s)=\frac{5.05}{s^2+2.15s+0.87}Q(s)-\frac{5.05}{1.01s+1}F(s)$$

$$H_2(s)=\frac{ab/aA_2}{s^2+2\frac{(a+b)}{2\sqrt{ab}}\sqrt{ab}\,s+ab}Q(s)-\frac{1/aA_2}{\frac{s}{a}+1}F(s)$$

$$K=\frac{2\sqrt{h_{20}}}{k_2}=5.05 \qquad \tau=\frac{2A_2\sqrt{h_{20}}}{k_2}=1.01 \qquad \omega_n^2=\frac{k_1k_2}{4A_1A_2\sqrt{h_{10}h_{20}}}=0.87$$

$$\delta=\frac{k_2A_1\sqrt{h_{10}}+k_1A_2\sqrt{h_{20}}}{\sqrt{k_1k_2}}\frac{1}{2\sqrt{A_1A_2\sqrt{h_{10}h_{20}}}}=1.16$$

Identificación por respuesta salto



Dos experimentos independientes:

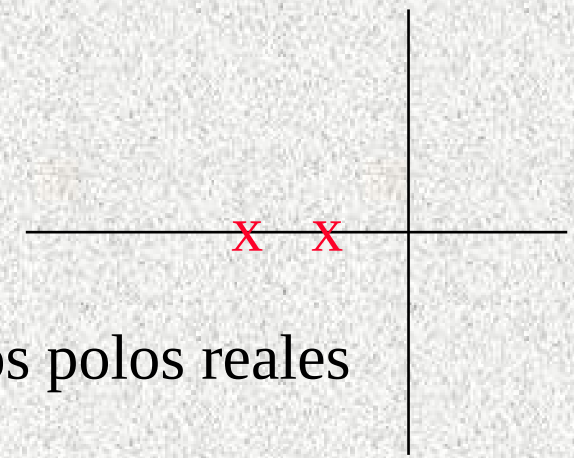
- Cambio en q con F cte.
- Cambio en F con q cte.

Ajuste con funciones de primer orden

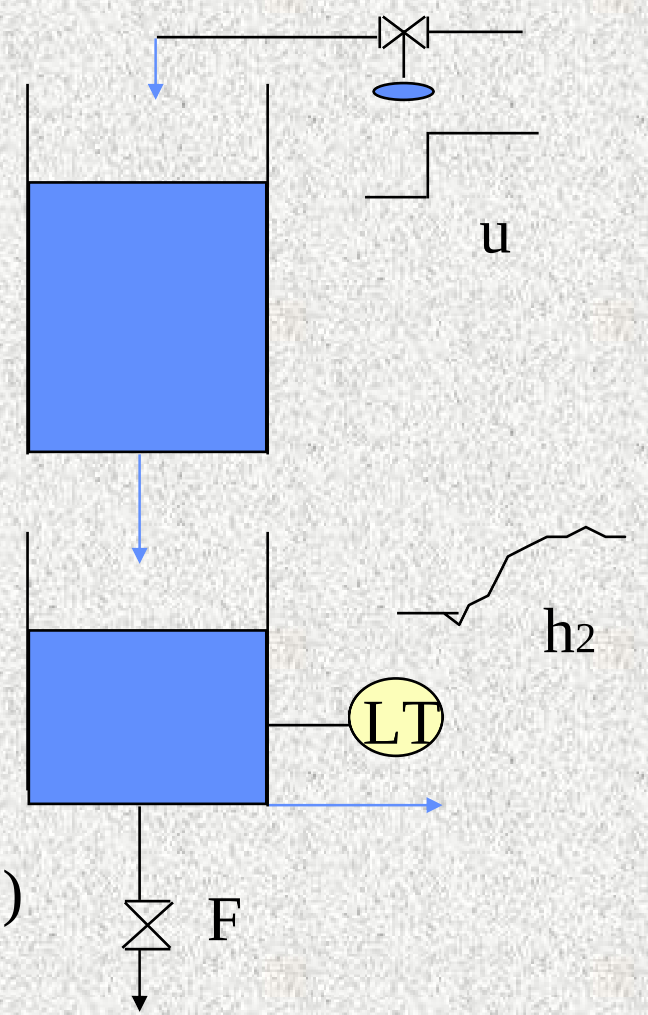
$$H_2(s) = \frac{K_q e^{-ds}}{\tau_q s + 1} U(s) = \frac{0.127 e^{-0.71s}}{1.64s + 1} U(s)$$

$$H_2(s) = \frac{K_f}{\tau_f s + 1} F(s) = \frac{-0.5}{0.99s + 1} F(s)$$

Análisis en lazo abierto

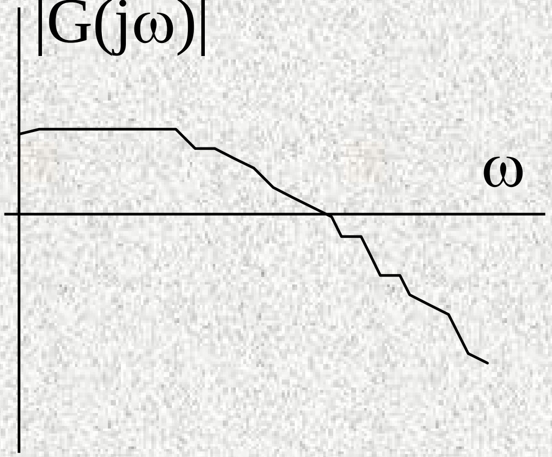


Dos polos reales



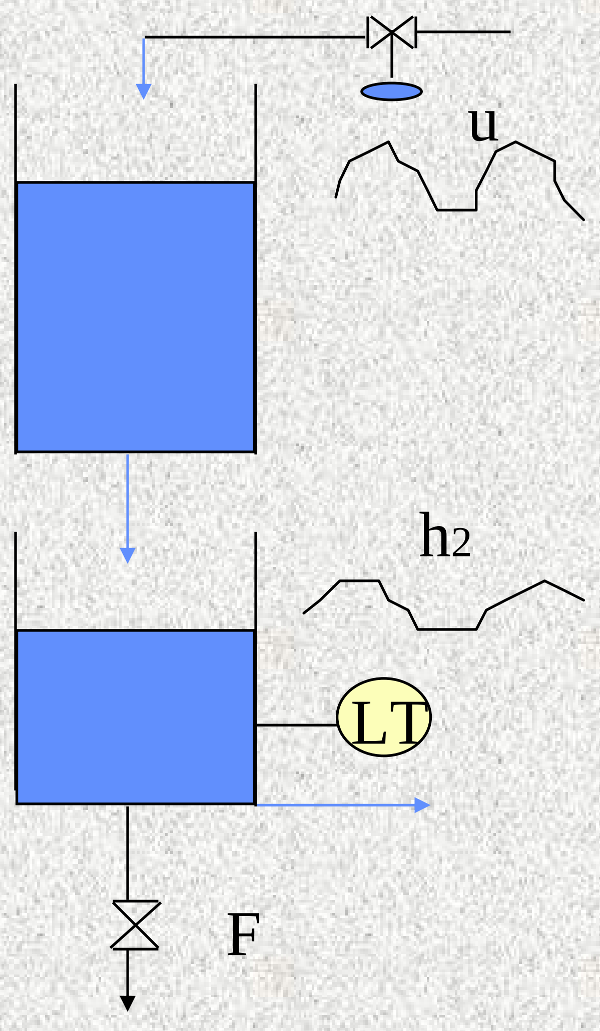
$$H_2(s) = \frac{0.126}{(1.01s+1)(1.14s+1)} U(s) - \frac{0.505}{1.01s+1} F(s)$$

Respuesta en frecuencia

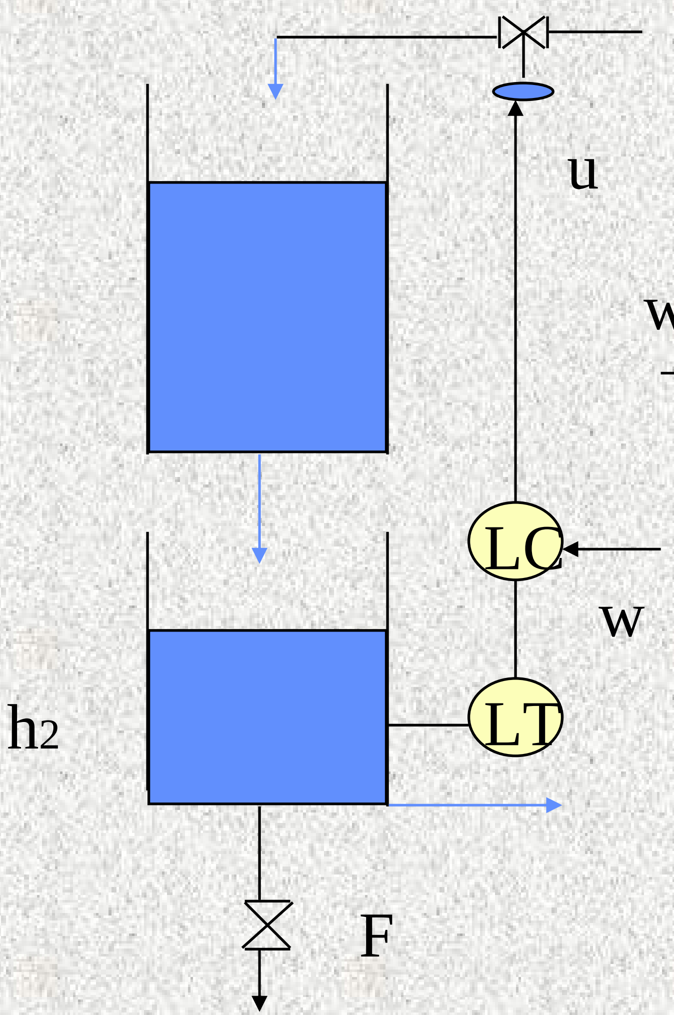


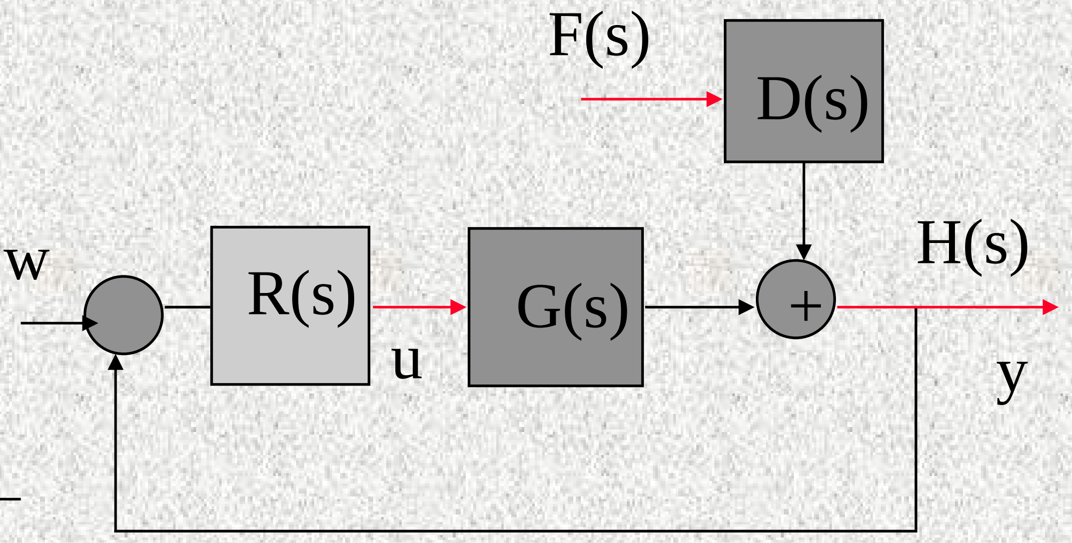
$$G(j\omega) = \frac{0.126}{(1.01j\omega + 1)(1.14j\omega + 1)}$$

$$D(j\omega) = \frac{-0.505}{1.01j\omega + 1}$$



Análisis en lazo cerrado

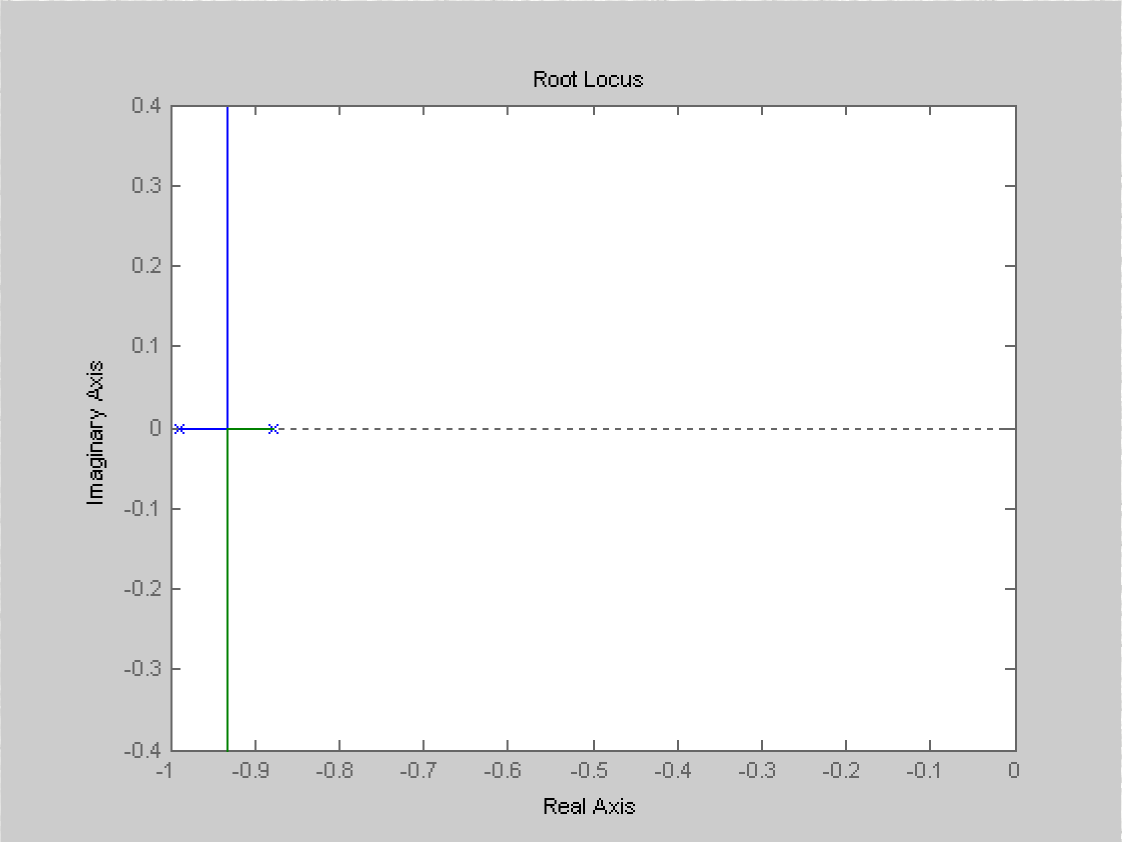




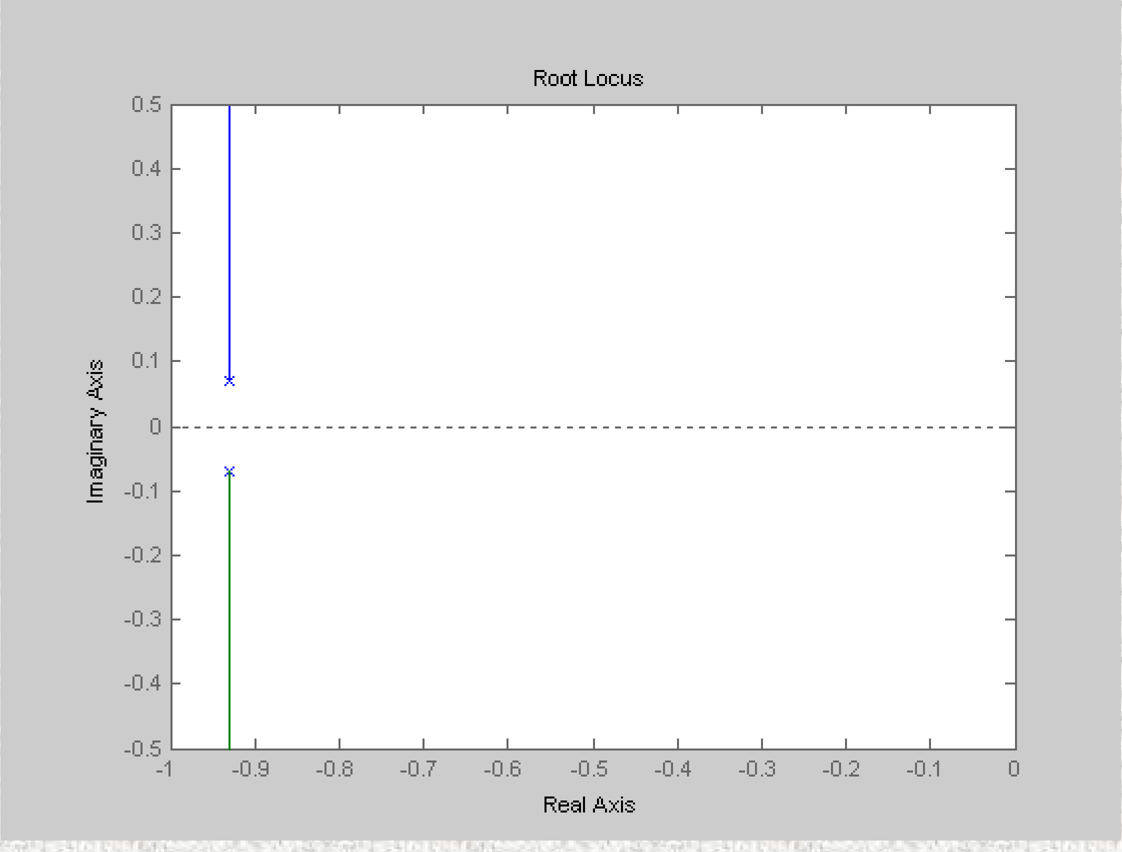
$$H(s) = \frac{G(s)R(s)}{1+G(s)R(s)}W(s) + \frac{D(s)}{1+G(s)R(s)}F(s)$$

Lugar de las raices

$1 + K_p G(s) = 0$



Precaución con los polinomios



Un cambio de

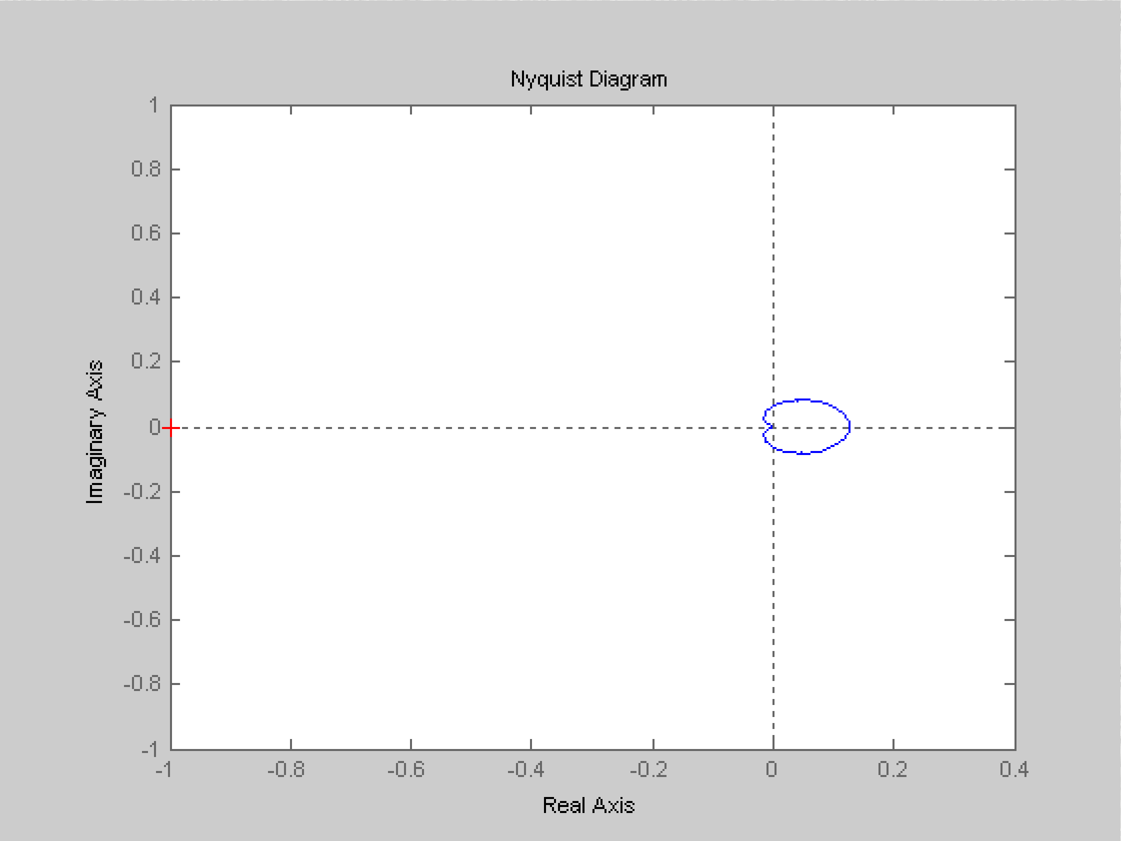
$$\frac{0.11}{s^2 + 1.8673s + 0.8685}$$

a

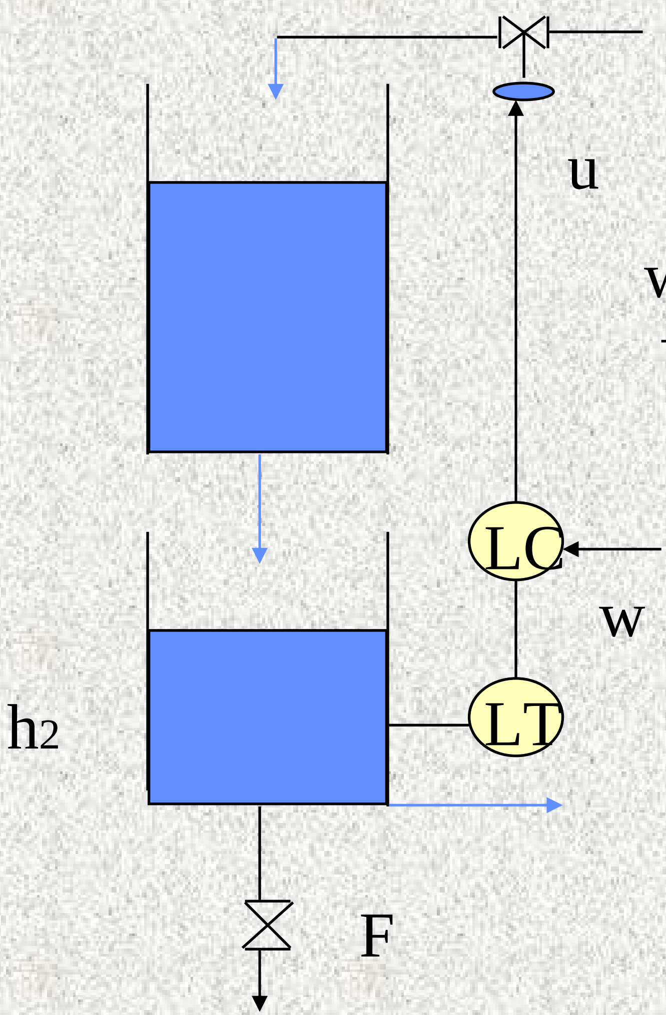
$$\frac{0.11}{s^2 + 1.86s + 0.87}$$

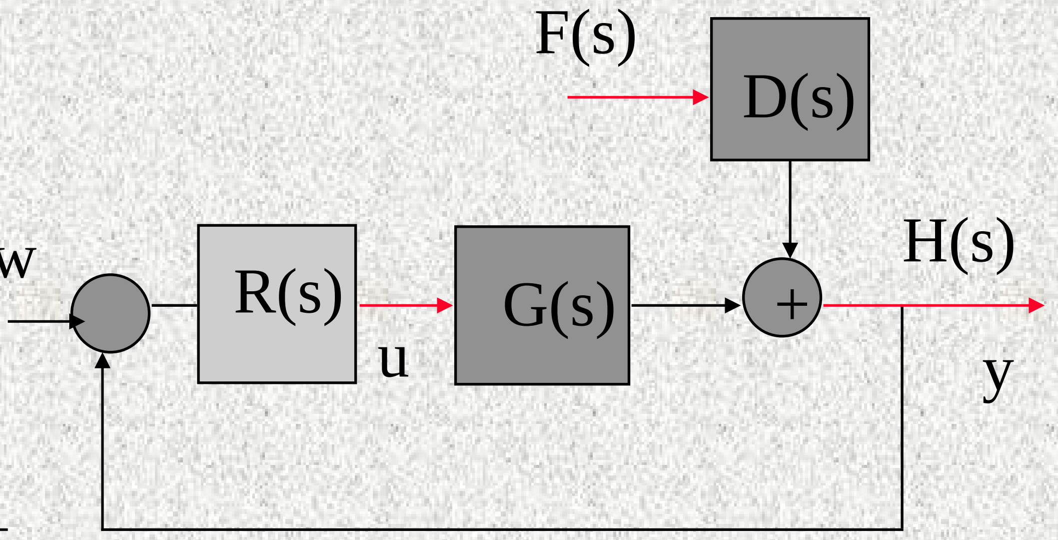
Da un lugar de las raices distinto!

Respuesta en frecuencia



Sintonía de un regulador





Criterio: minimizar el error acumulado ante cambios en la referencia y no tener error estacionario

PI o PID criterio de Rovira

Tabla de Rovira y otros

PI paralelo

Criterio	Proporcional	Integral	Derivativo
MIAE	a=0.758	a=-0.323	
	b=-0.861	b=1.020	
MITAE	a=0.586	a=-0.165	
	b=-0.916	b=1.030	

PID Paralelo

Criterio	Proporcional	Integral	Derivativo
MIAE	a=1.086	a=-0.130	a=0.348
	b=-0.869	b=0.740	b=0.914
MITAE	a=0.965	a=-0.147	a=0.308
	b=-0.855	b=0.796	b=0.929

$$K_p K = a\left(\frac{d}{\tau}\right)^b$$

$$\frac{\tau}{T_i} = a\left(\frac{d}{\tau}\right) + b$$

$$\frac{T_d}{\tau} = a\left(\frac{d}{\tau}\right)^b$$

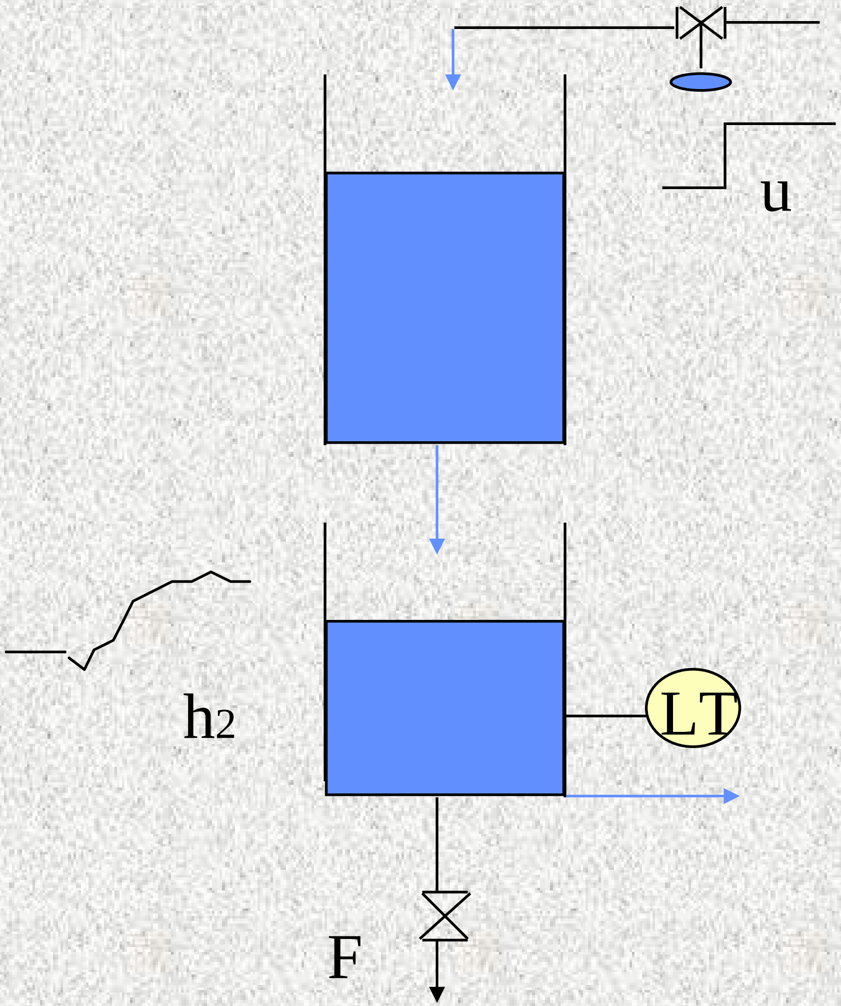
K en % / %

Sintonia para cambios de consigna

Validas para procesos monótonos con $d / \tau < 1$

Para reguladores digitales aumentar d en medio periodo de muestreo

Identificación por respuesta salto



Experimento:
Cambio en u con F cte.

$$H_2(s) = \frac{0.127e^{-0.71s}}{1.64s+1}U(s)$$

si el transmisor de nivel esta calibrado en el rango 0 - 8.5 m.:

$$H_2(s) = \frac{0.127\,100/8.5e^{-0.71s}}{1.64s+1}U(s) =$$

$$= \frac{1.49e^{-0.71s}}{1.64s+1}U(s)$$

Sintonía

PI:

$$K_pK = a\left(\frac{d}{\tau}\right)^b \qquad K_p = \frac{0.758}{1.49}\left(\frac{0.71}{1.64}\right)^{-0.861} = 1.05\%/\% = 12.35\%/m$$

$$\frac{\tau}{T_i} = a\left(\frac{d}{\tau}\right) + b \qquad T_i = \frac{1.64}{-0.323\left(\frac{0.71}{1.64}\right) + 1.02} = 1.86\,\text{min}$$

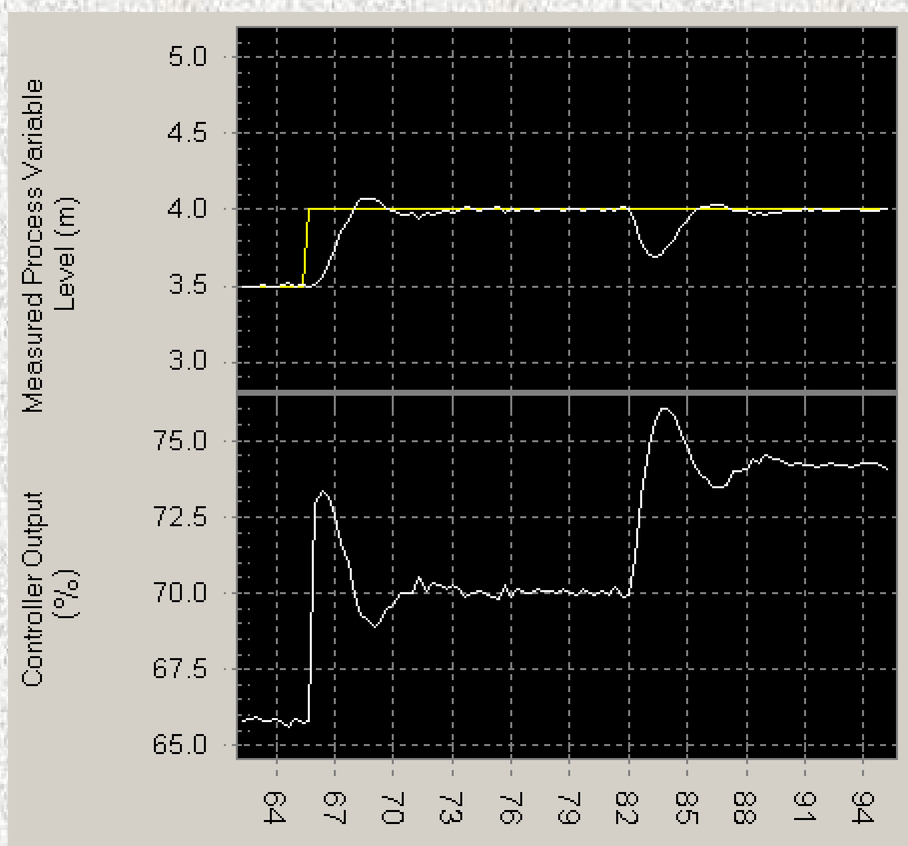
PID:

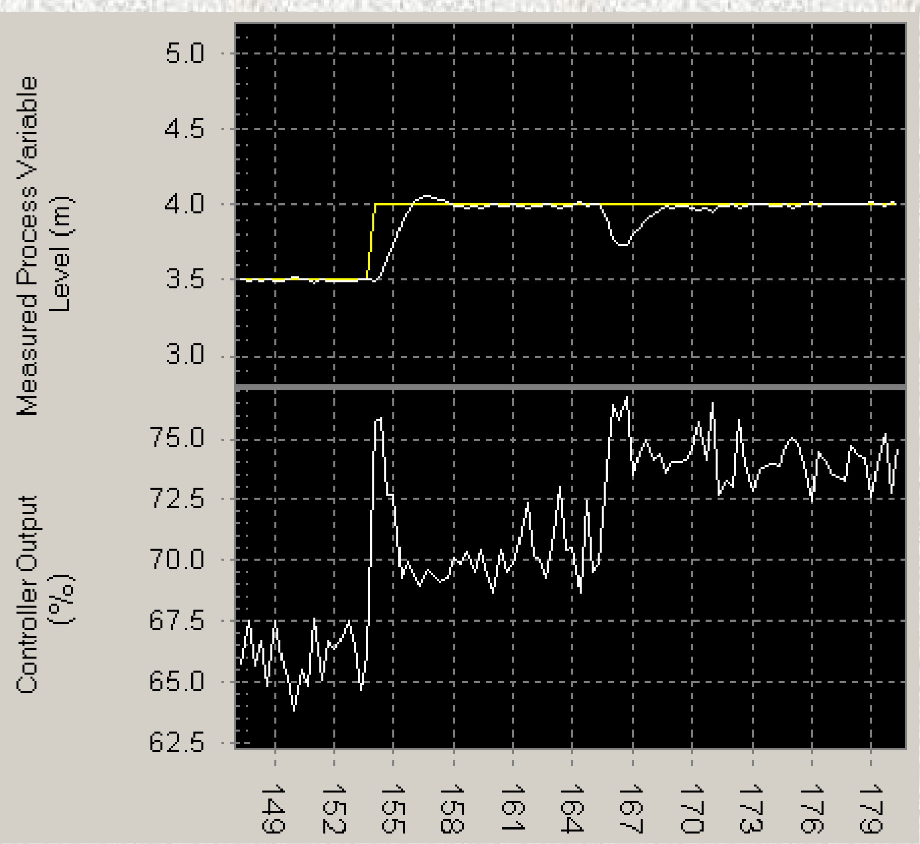
$$K_pK = a\left(\frac{d}{\tau}\right)^b \qquad K_p = \frac{1.086}{1.49}\left(\frac{0.71}{1.64}\right)^{-0.869} = 1.51\%/\% = 17.75\%/m$$

$$\frac{\tau}{T_i} = a\left(\frac{d}{\tau}\right) + b \qquad T_i = \frac{1.64}{-0.13\left(\frac{0.71}{1.64}\right) + 0.74} = 2.4\,\text{min}$$

$$\frac{T_d}{\tau} = a\left(\frac{d}{\tau}\right)^b \qquad T_d = 1.64\;0.348\left(\frac{0.71}{1.64}\right)^{0.914} = 0.26\,\text{min}$$

Rovira MIAE





PI respuesta ante cambios de referencia y perturbaciones

PID respuesta ante cambios de referencia y perturbaciones

Otros criterios

Rechazo de perturbaciones minimizando el error acumulado y sin error estacionario

Criteria	Proportional	Integral	Derivative
MIAE	a=0.984 b=-0.986	a=0.608 b=-0.707	
MISE	a=1.305 b=-0.959	a=0.492 b=-0.739	
MITAE	a=0.859 b=-0.977	a=0.674 b=-0.68	

$$K_p K = a\left(\frac{d}{\tau}\right)^b$$

$$\frac{\tau}{T_i} = a\left(\frac{d}{\tau}\right)^b$$

$$\frac{T_d}{\tau} = a\left(\frac{d}{\tau}\right)^b$$

Puede usarse un PI con el método de sintonía de Lopez y criterio MIAE

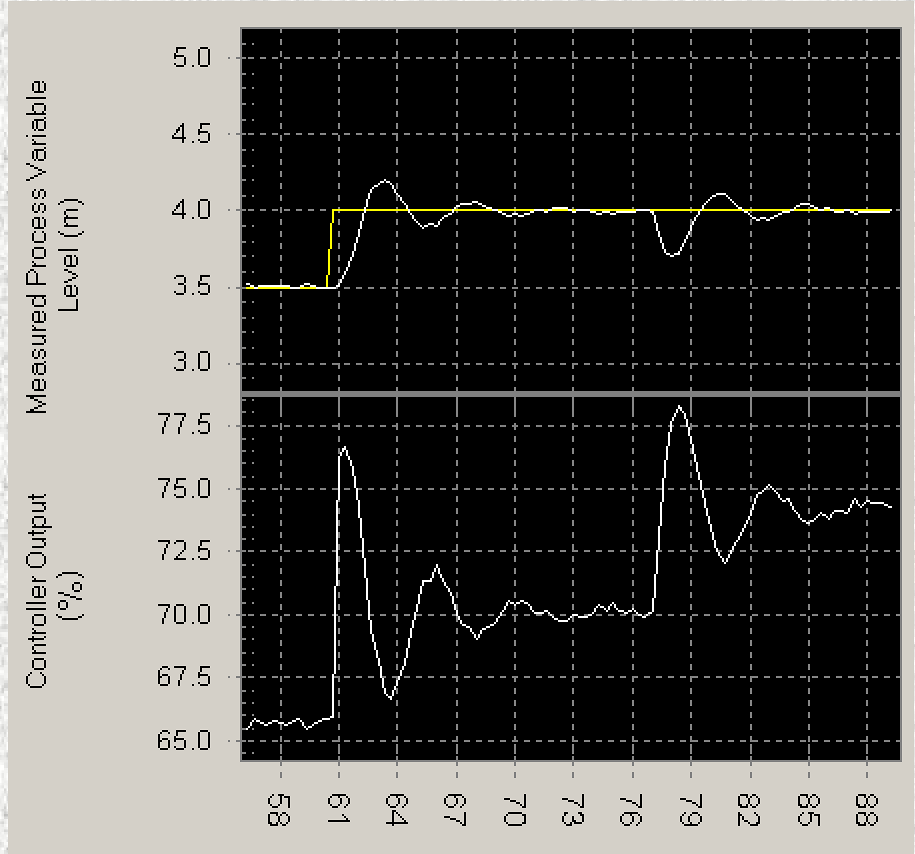
Lopez MIAE

$$K_p K = a\left(\frac{d}{\tau}\right)^b$$

$$K_p = \frac{0.984}{0.127}\left(\frac{0.71}{1.64}\right)^{-0.986} = 17.37\% / m$$

$$\frac{\tau}{T_i} = a\left(\frac{d}{\tau}\right)^b$$

$$T_i = \frac{1.64}{0.608\left(\frac{0.71}{1.64}\right)^{-0.707}} = 1.49\,\text{min}$$



Respuesta ante cambios en la referencia y perturbaciones

Lopez MITAE

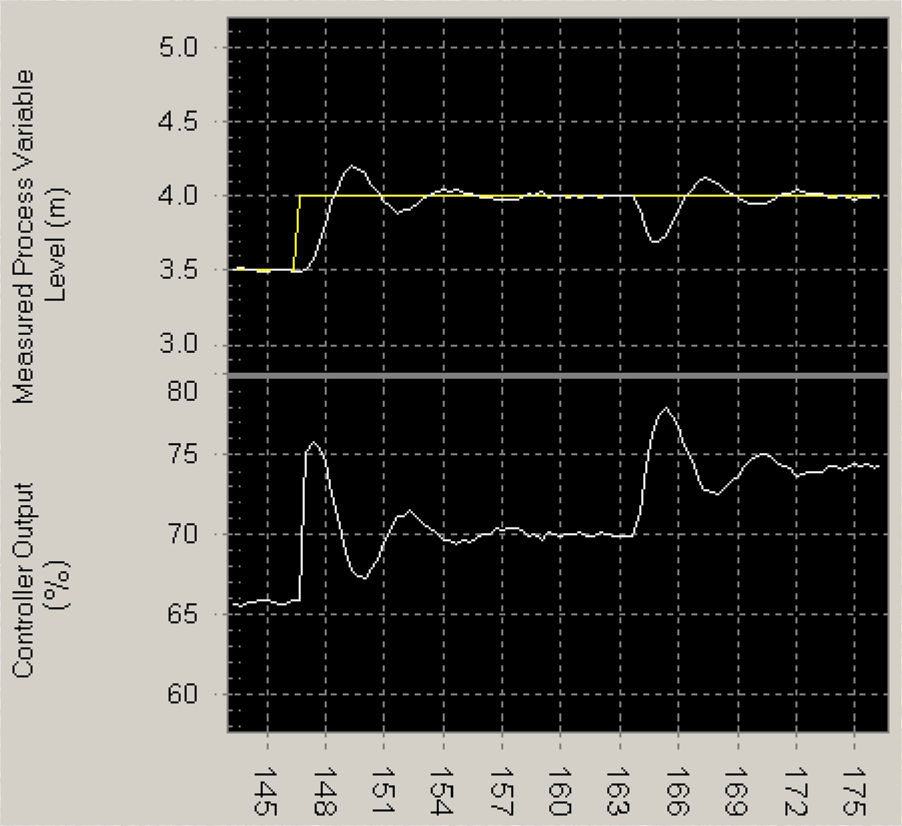
$$K_p K = a\left(\frac{d}{\tau}\right)^b$$

$$K_p = \frac{0.859}{0.127}\left(\frac{0.71}{1.64}\right)^{-0.977} = 15.32\% / m$$

$$\frac{\tau}{T_i} = a\left(\frac{d}{\tau}\right)^b$$

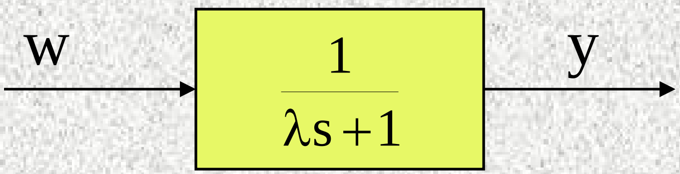
$$T_i = \frac{1.64}{0.674\left(\frac{0.71}{1.64}\right)^{-0.68}} = 1.377 \min$$

Respuesta ante cambios en la referencia y perturbaciones



Sintonía: λ Tuning

Tipo	K_p	T_i	T_d	λ recomendado λ>0.2τ siempre
PI	$\frac{\tau}{K\lambda}$	τ		$\frac{\lambda}{d} > 1.7$
PI mejorado	$\frac{2\tau + d}{2K\lambda}$	$\tau + \frac{d}{2}$		$\frac{\lambda}{d} > 1.7$
PID	$\frac{2\tau + d}{2K(\lambda + d)}$	$\tau + \frac{d}{2}$	$\frac{\tau d}{2\tau + d}$	$\frac{\lambda}{d} > 0.25$



λ constante de tiempo deseada en lazo cerrado

Sintonía

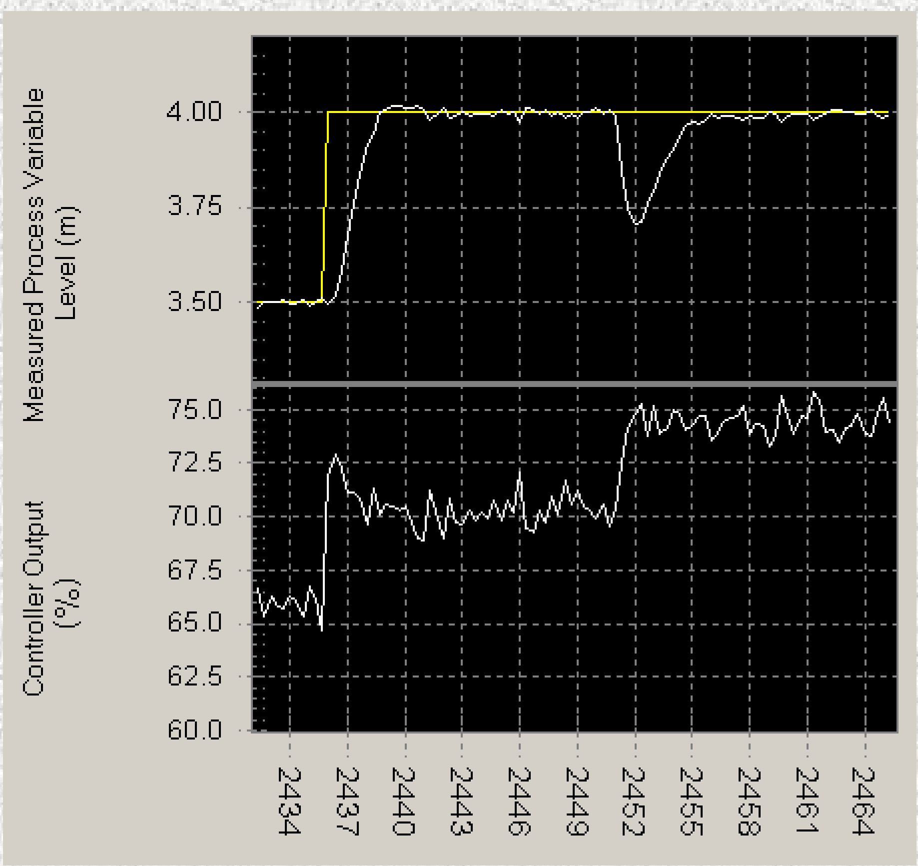
Criterio: Tiempo de asentamiento en lazo cerrado de 2 min ⇨ $\lambda = 0.66$ min cumple $\lambda / d > 0.25$

$$K_p = \frac{2\tau + d}{2K(\lambda + d)} =$$

$$= \frac{2\ 1.64 + 0.71}{2\ 0.127(0.66 + 0.71)} = 11.46\% / \text{m}$$

$$T_i = \tau + \frac{d}{2} = 1.64 + \frac{0.71}{2} = 1.99\,\text{min}$$

$$T_d = \frac{\tau d}{2\tau + d} = \frac{1.64\ \ 0.71}{2\ \ 1.64 + 0.71} = 0.29\,\text{min}$$



Respuesta a un cambio de consigna y a una perturbación

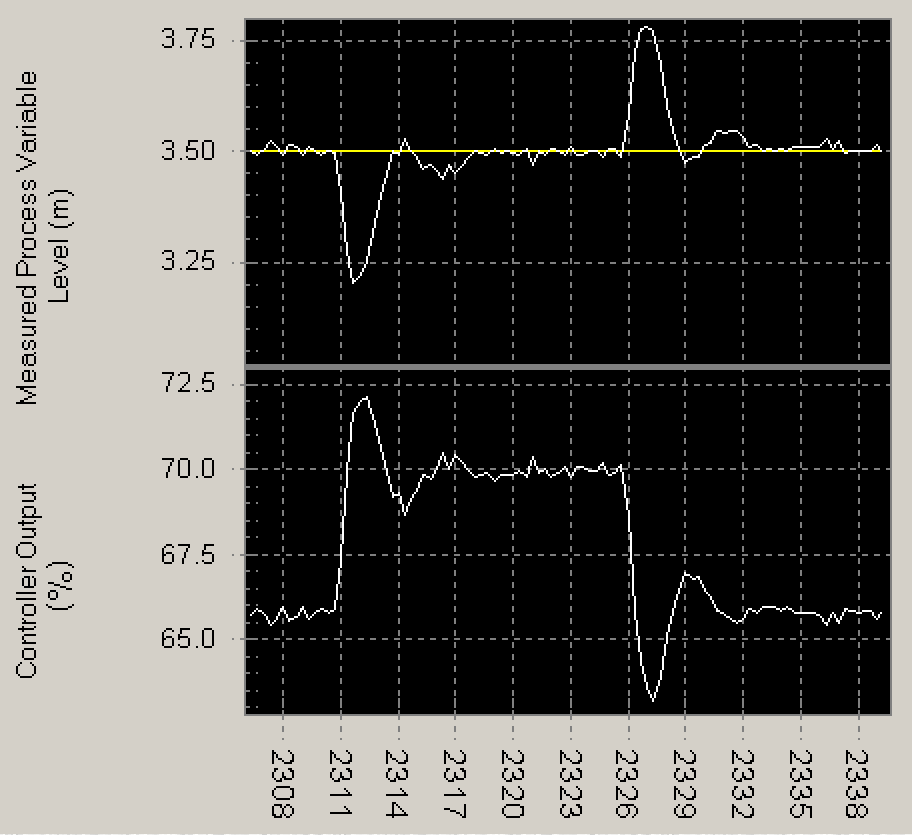
Ziegler Nichols

Type	**Gain K_p**	**Integral time**	**Derivative time**
P	τ / (K d)		
PI	0.9τ /(K d)	3.33 d	
Series PID	1.2τ /(K d)	2 d	0.5 d

$K_p = 0.9\tau/(Kd) = 0.9\ \ 1.64 /(0.127\ \ 0.71) = 16.37$

$T_i = 3.33\ d = 3.33\ 0.71 = 2.36$

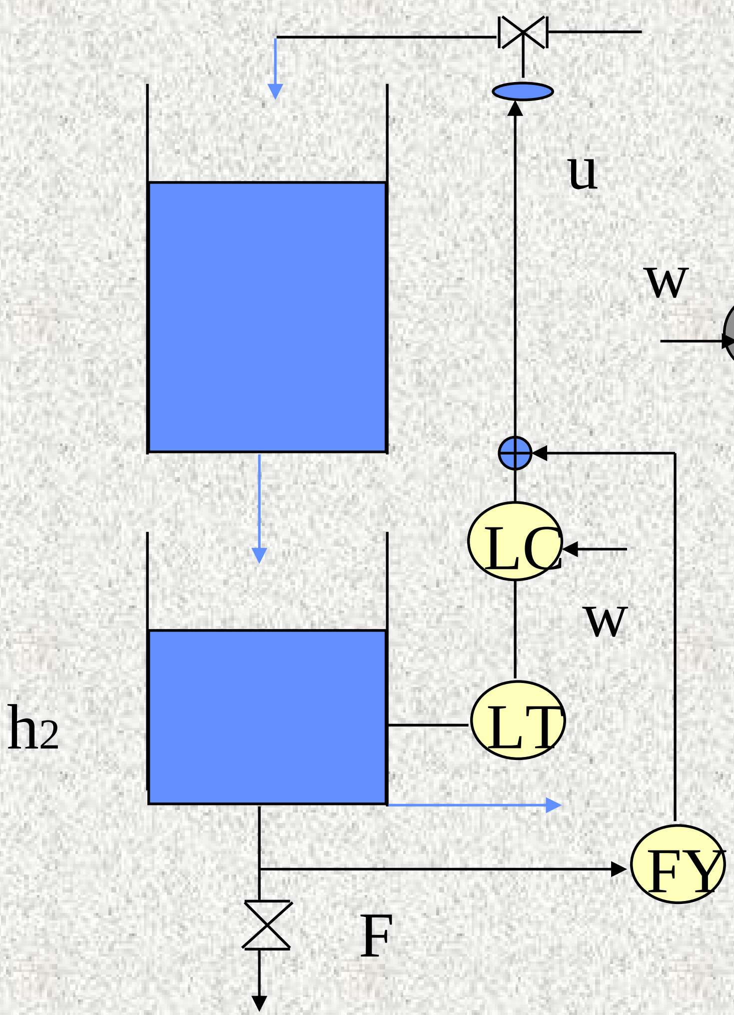
Ziegler-Nichols

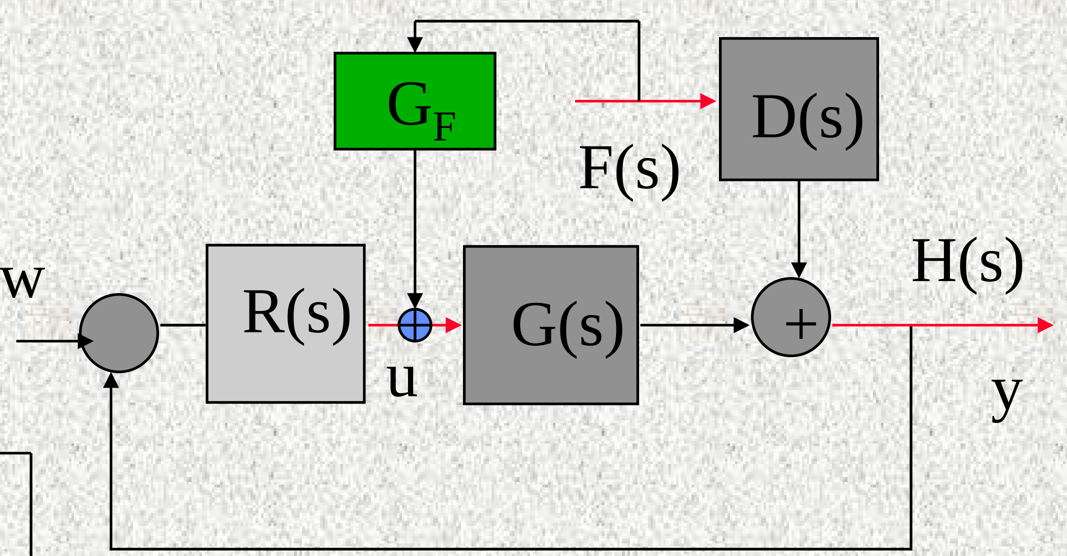


Respuesta a una perturbación. Verifica el criterio

Respuesta a un salto en la referencia

Feedforward





$$G_F = -\frac{G_P(s)}{G(s)}$$

Feedforward

$$H_2(s) = G(s)U(s) + D(s)F(s) = \frac{0.126}{(1.01s+1)(1.14s+1)}U(s) - \frac{0.505}{1.01s+1}F(s)$$

$$G_F = -\frac{G_P(s)}{G(s)} = \frac{0.505(1.01s+1)(1.14s+1)}{0.126(1.01s+1)} = 4(1.14s+1)$$

Una compensacion perfecta no es realizable físicamente al ser el proceso mas lento frente a cambios en U que frente a cambios en F, ni esta indicado un compensador en adelanto