Introduction to Modelling and Simulation

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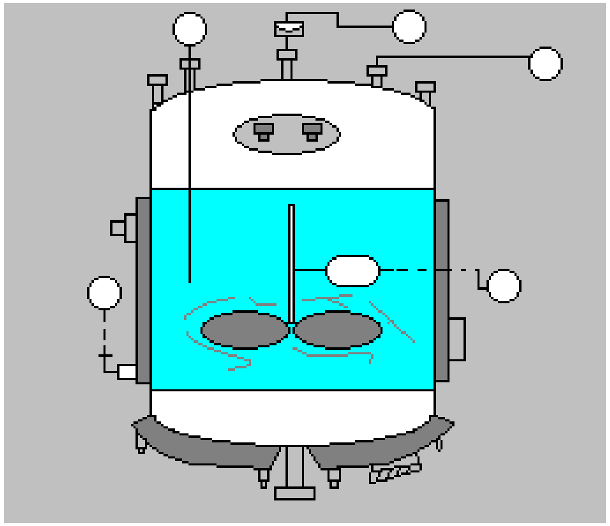
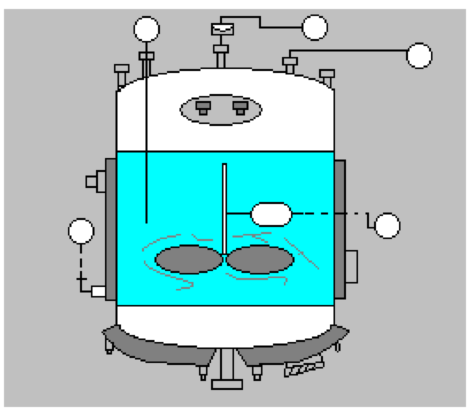
Digital simulation

- Methods and tools oriented to “imitate” or predict the responses of a systems against certain changes or “stimulus” using a computer.
- Modern name: digital twins



Modelling and Simulation

Capture the behaviour of key variables of a process by means of mathematical relations and solve them in a computer with a specific purpose

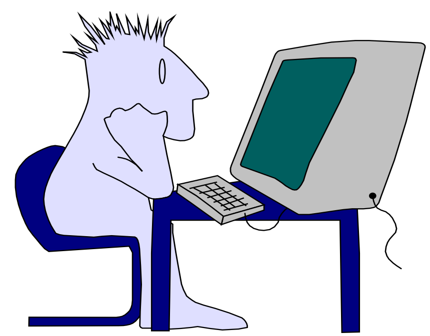


Process

$$\frac{dx}{dt} = f(x, u, p)$$

$$y = g(x, u)$$

Model



Simulation

Aims of the simulation

- Study of a process, what if…? analysis
- Design (process, control,…)
- Testing a control system before actual implementation in the plant
- Personnel training
- Operation optimization
- Essays in a virtual plant …..
- Decision support
- Digital Twin (real-time, simulation fed with plant data running in parallel to the real process)

Advantages of using simulation

Performs changes that, if implemented in the process, will be

- Very costly,
- Too slow / fast
- dangerous, etc.

Reproduces the experiment as many times as desired under the same conditions

Saves time

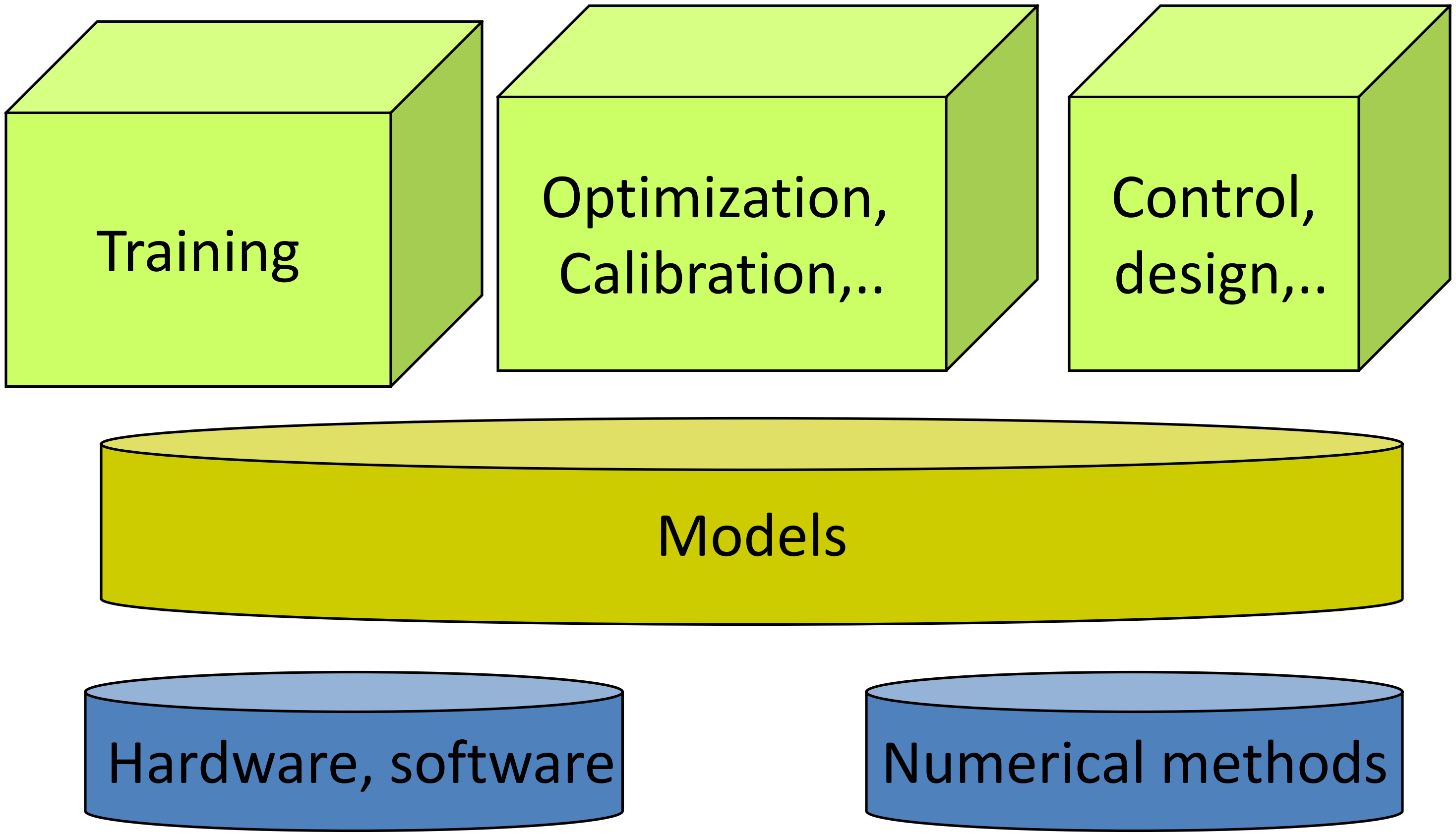
Provides safety

Allows sensitivity studies

Provides a model that can be used for many purposes

Allows experimenting with systems that are not built yet

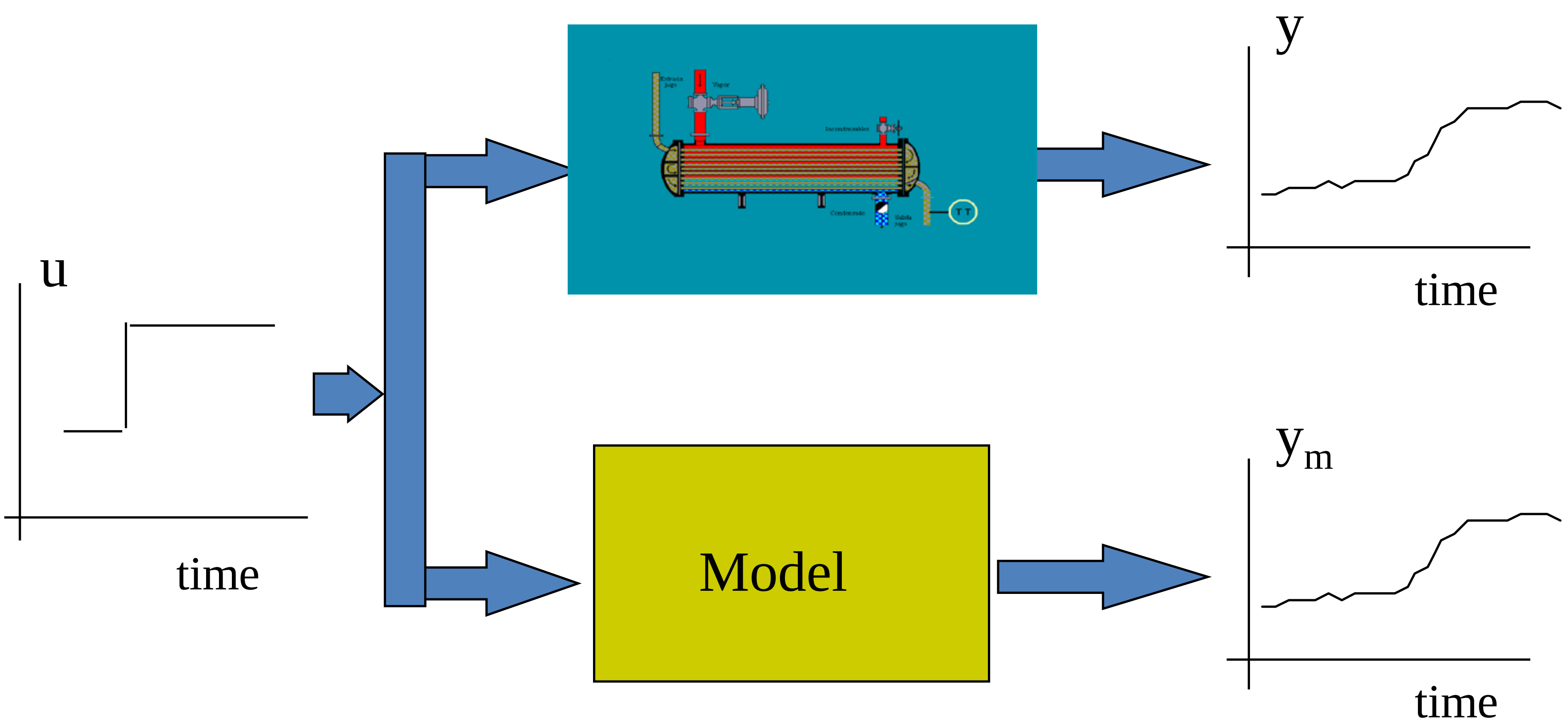
Simulation environment



Models

- Simulation is based on mathematical models of the processes.
- Mathematical models are set of equations relating the variables of a process and being able to provide an adequate representation of its behaviour.
- They are always approximations of the real world
- Adequacy of a model depends on their intended use: It is a balance between accuracy in the representation of the real world and easiness in the mathematical handling of the model
- There are a wide variety of models according to the type of processes they represent and their aims.

Adequate representation



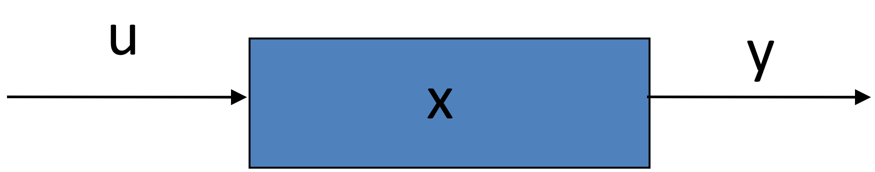
+ adequacy and facility of use in the intended application

State space models

$$\frac{d\,x(t)}{dt} = f\,(x(t), u(t), t)$$

$$y(t) = g(x, u(t), t)$$

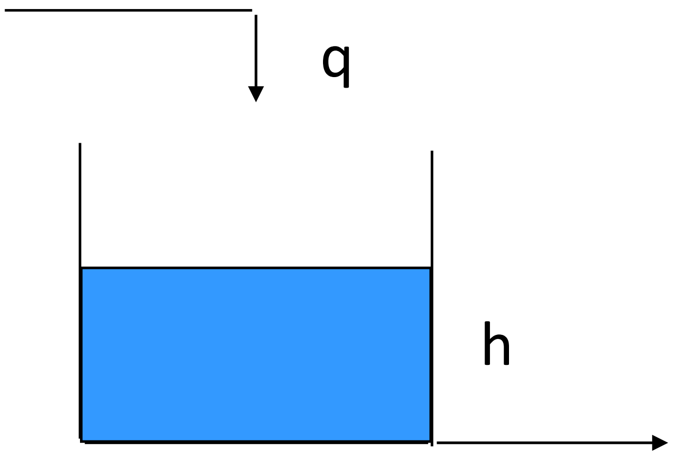
Manipulated variables and disturbances



Model responses

x States

Continuous versus discrete event systems



Continuous processes:
Their variables evolve continuously in time and can have any value within a certain range

Discrete event systems:
Their variables only change values at certain time instants and they take values only from a discrete set of values

Continuous versus discrete event systems

- Continuous processes:
  - Described usually by means of DAEs or PDEs.
  - Main interest: Compute the trajectory of some variables
- Discrete events processes:
  - Described usually as sequences of activities.
  - Main interest: Statistical behavior of some variables

Discrete events simulation

- Oriented to describe and solve the evolution of processes described by a set of ordered activities (activity network) , and provide tools for analyzing the statistical behavior of some variables
- Examples:
  - Single objects Manufacturing (shoes, cars,..)
  - Maintenance systems
  - Offices
  - Hospitals
  - Services, ….

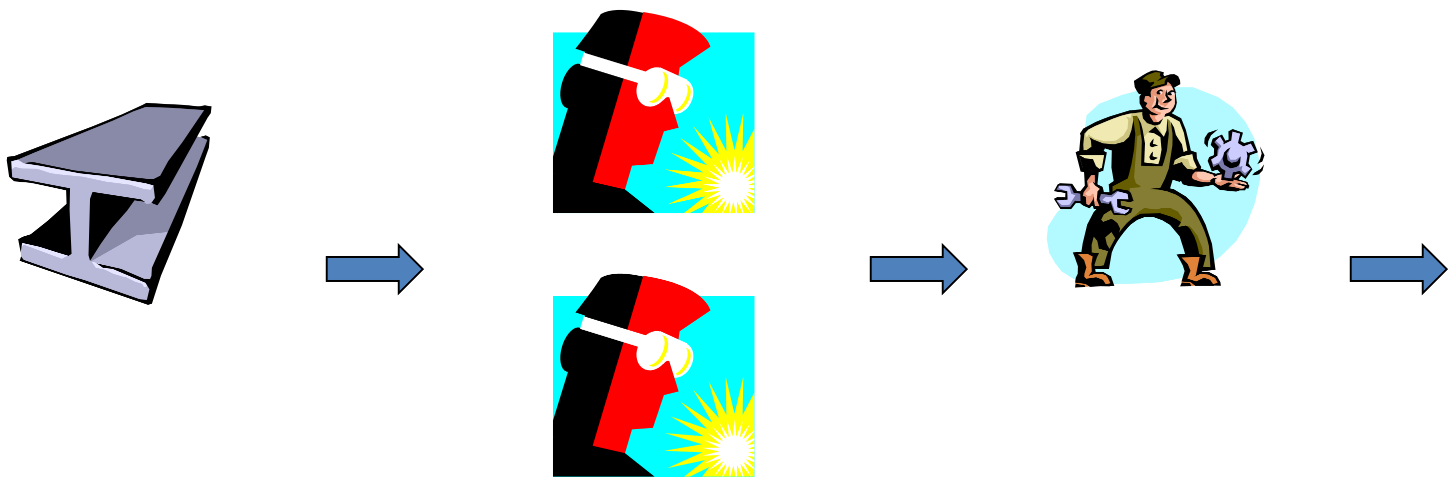
Example: Telephone switchboard

Calls arrive with a certain statistical distribution. If there is no line available, the call is retain waiting to be served.

Which is the average waiting time?

How many lines should be installed, so that the average waiting time is less than a given threshold?

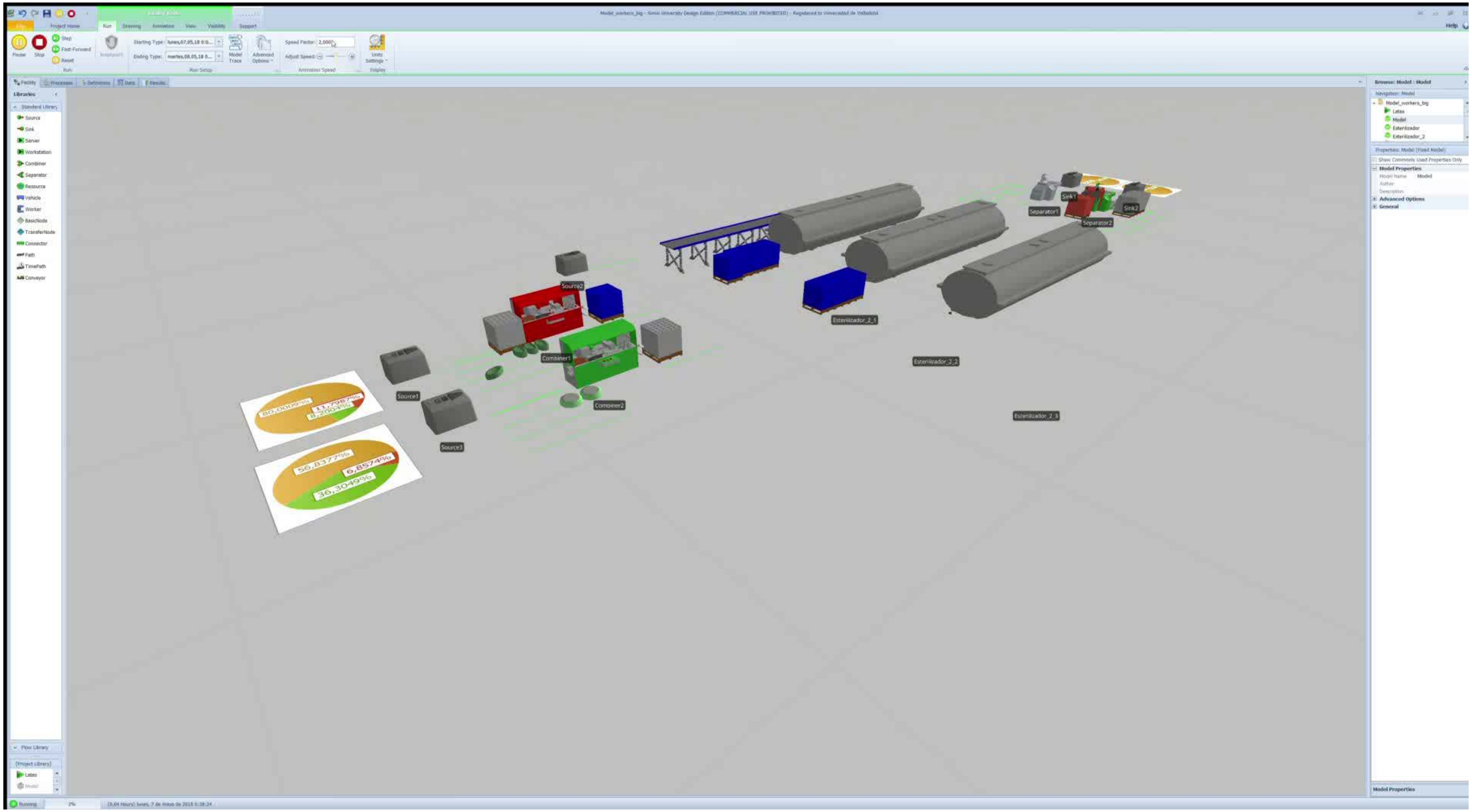
Example: Manufacturing shopflow



The products follow a set of stages, with a certain elapsed time in each machine , queuing to be treated until the next machine id free.

Which is the average length of each queue? How many machines should be installed in parallel at each stage to keep the average queue length below a certain value?

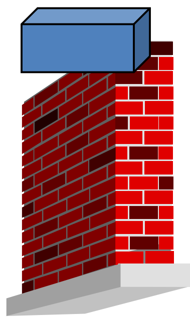
Tuna canning factory

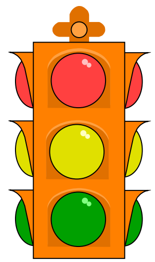


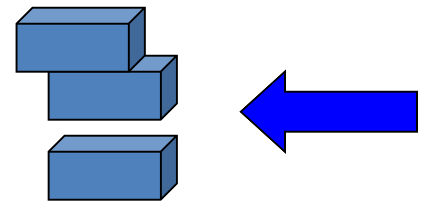
Main elements of discrete-events simulation

- Time
- Sequence of tasks
- Events that activate the tasks or are activated by the tasks
- Entities involved in the tasks
- Resources required, generated or consumed in the tasks
- Waiting queues, storage rooms, etc.

Main elements of discrete-events simulation







Tasks:

load, move, unload…

Entities:

boxes

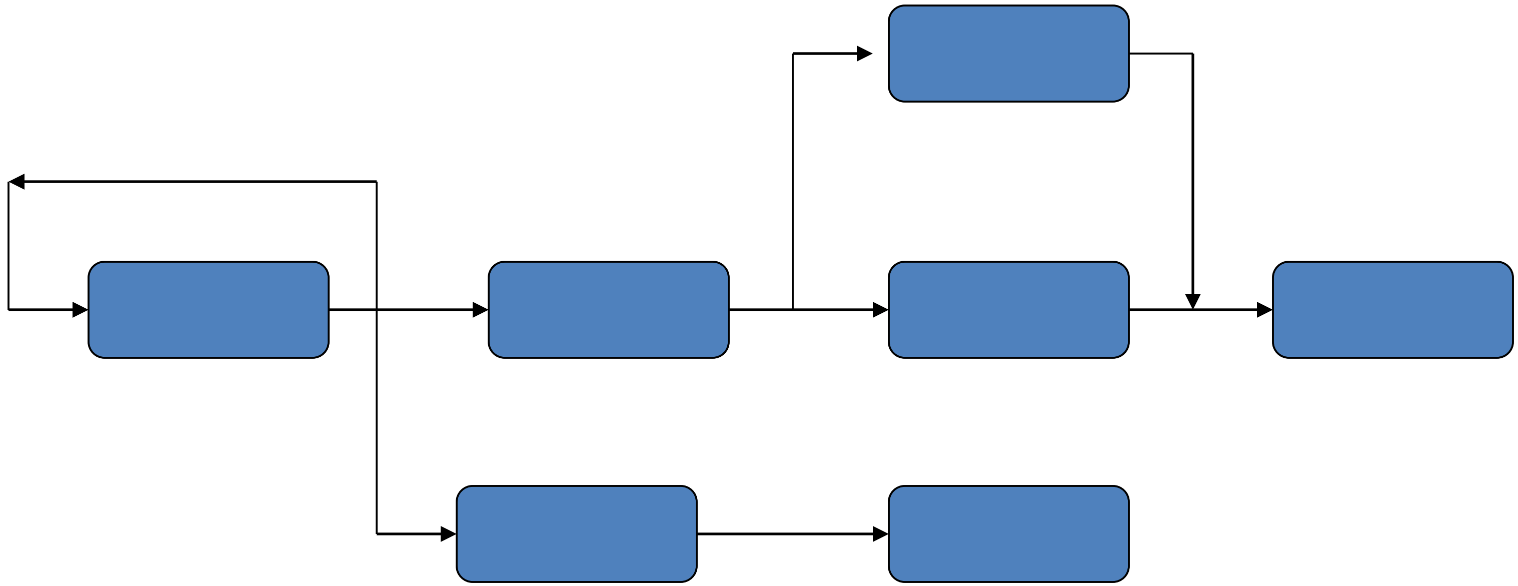
Resources:

People, trucks,..

Events:

Boxes arrival,
Green/Red light
truck busy,...

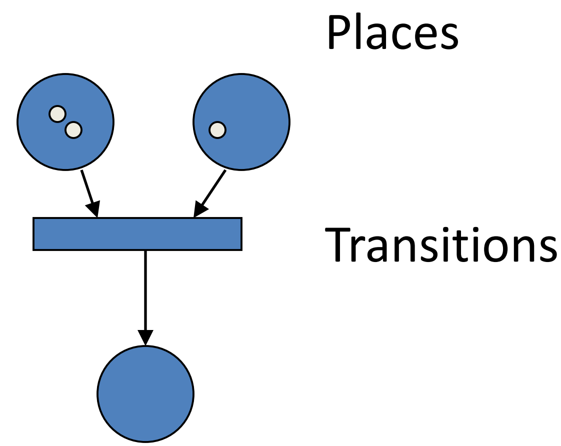
Activity network



Ordered sequence of tasks activated by time or event

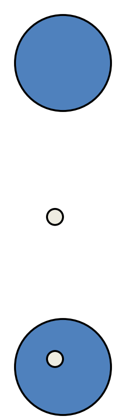
Discrete-Events models

Description of the set of logic relations governing the interactions among the systems elements



Petri Nets

GRAFCET

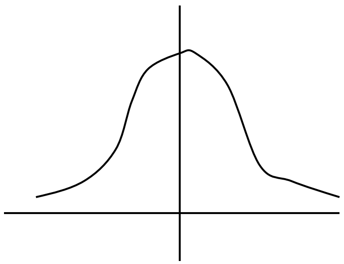


Queues, state conditions,...

Resources

Condition fulfilled

Discrete-events simulation



Task network

Boundary conditions

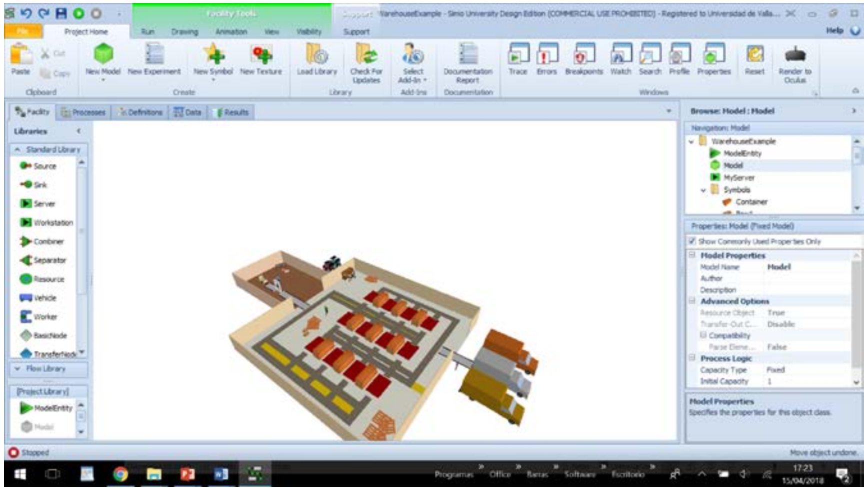
Results

Executed N times according to the statistical distribution given by the boundary conditions

Results of the N simulations are analysed in order to infer its statistical properties

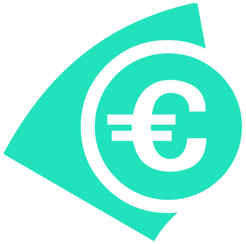
Software

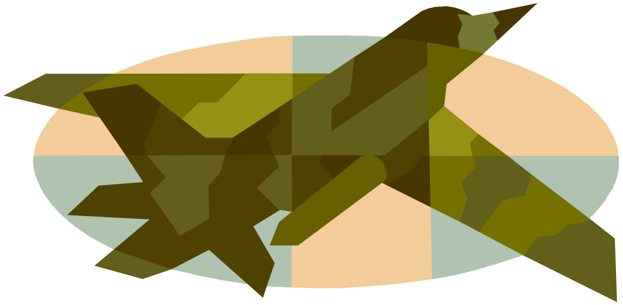
MicroSaint, Witness, ARENA
SIMIO

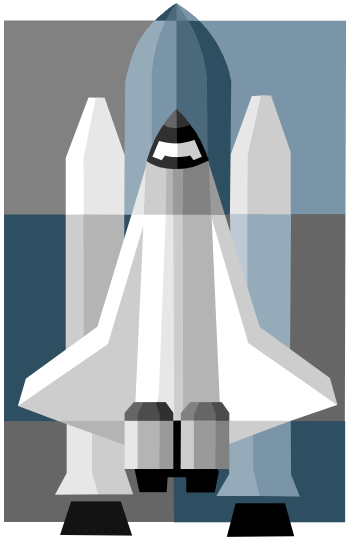


- Animation
- Useful for debugging
- Important for presentation and explotation of the simulation
- Requires time and resources

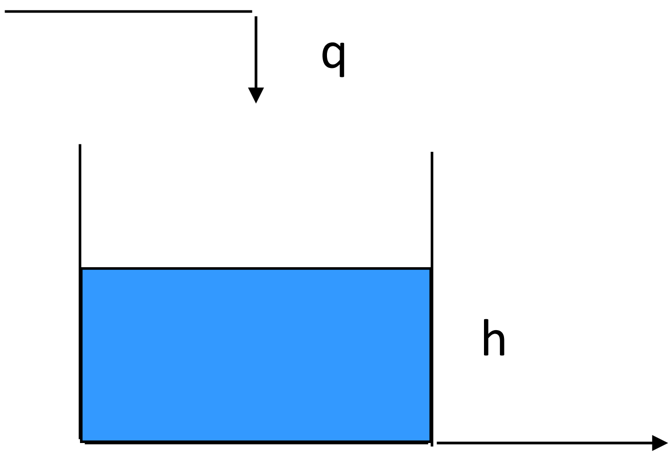
Continuous processes







Static and dynamic models



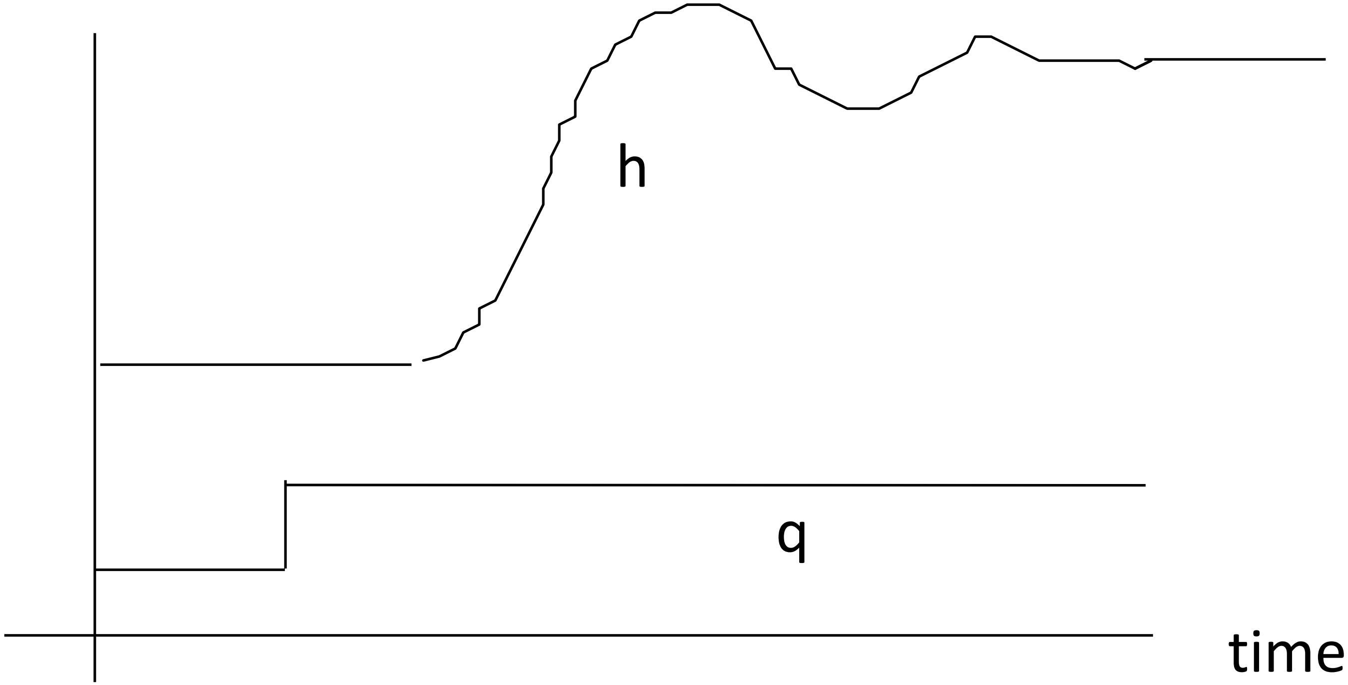
$$q = k\rho\sqrt{h}$$

Static model:
Gives relations among the variables corresponding to an equilibrium state

$$A\rho\frac{dh}{dt} = q - k\rho\sqrt{h}$$

Dynamic model:
Gives relations among the variables over time

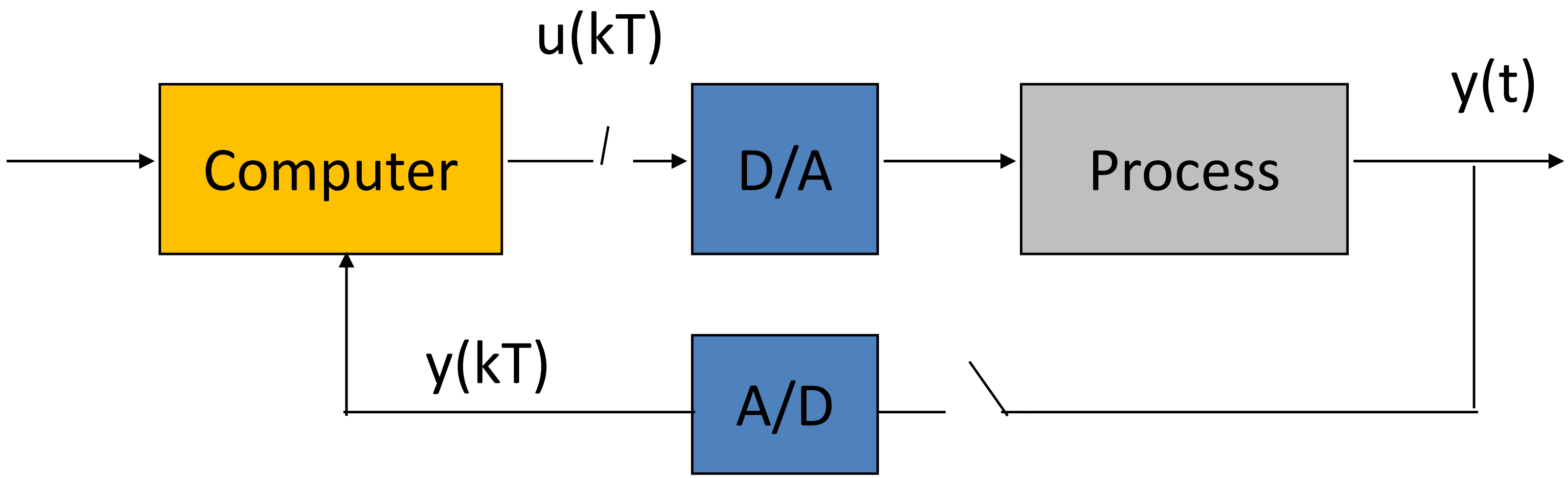
Dynamic response



Static and dynamic models

- Static models:
  - Describe systems in equilibrium
  - Algebraic equations
  - Design oriented
- Dynamic models in continuous time
  - Describe the time evolution of a system
  - ODEs, DAEs and PDEs
  - General purpose use

Sampled data models

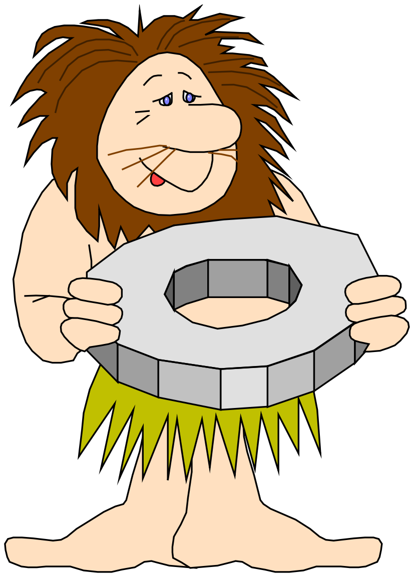


- ✓ Discrete time models
- ✓ They relate the process variables at the sampling times kT
- ✓ Equations in differences $y((k+1)T) = f(y(kT), u(kT))$

$K = 0, 1, 2, \ldots$

Methods to obtain models

Using knowledge,
nature laws,
reasoning,…

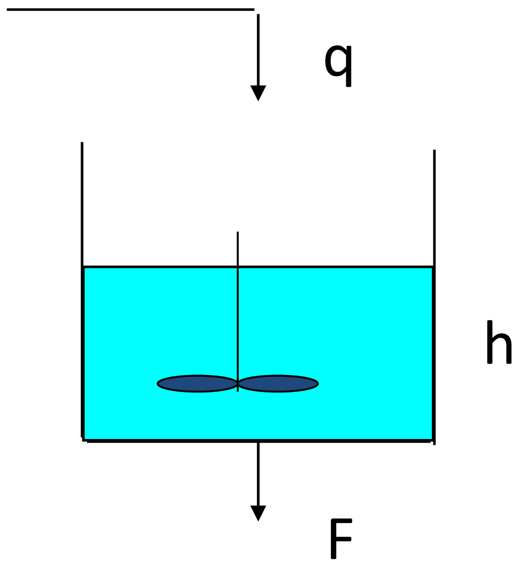


Using experimentation
and data analysis

First principles models

- There are obtained using knowledge on the system and reasoning based on the application of laws of nature (mass and energy balances, laws of the application, domain, …)
- There are based on a set of hypothesis on the system
- They have general validity if the hypothesis are respected
- Development of the models requires good knowledge of the system and good command of the physical laws.

Example: Tank



Mass conservation

Accumulation=
inflow q – outflow F

$$\frac{dm}{dt} = q\rho - F\rho$$

$$m = A\rho h \qquad F = k\sqrt{h}$$

$$A\frac{dh}{dt} = q - k\sqrt{h} \qquad V = Ah$$

Non-linear differential equation

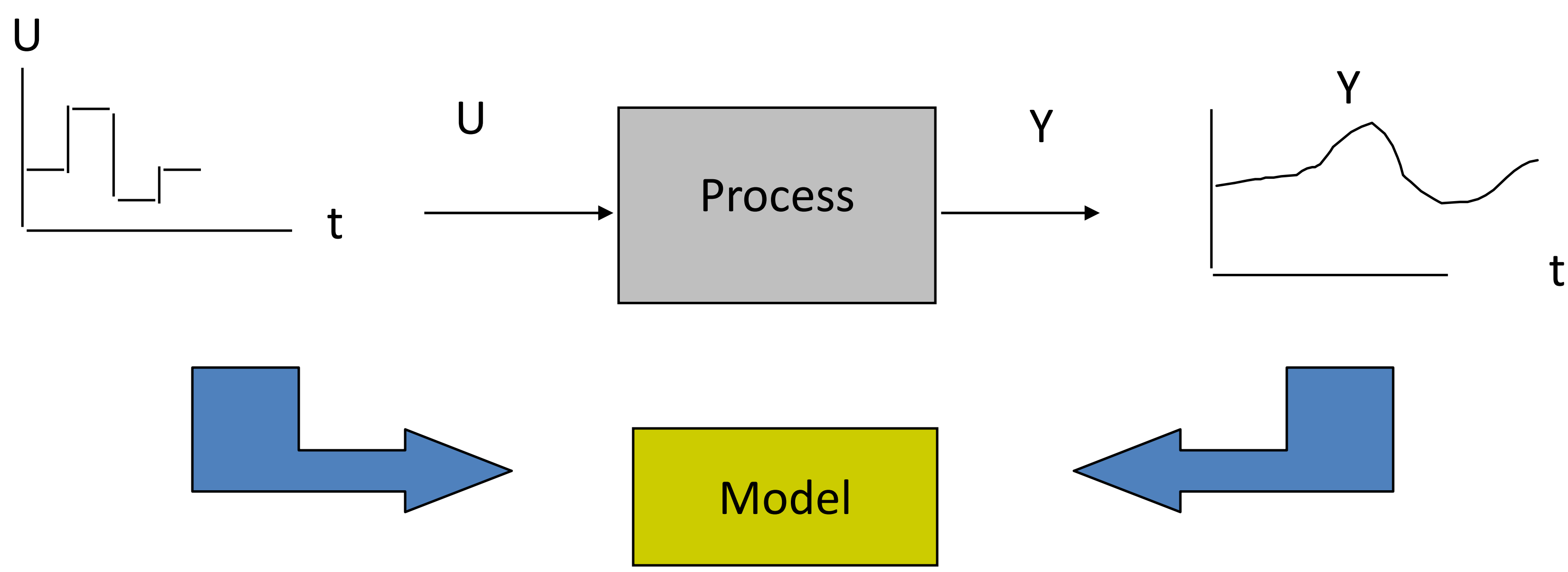
Algebraic equation

Parameters

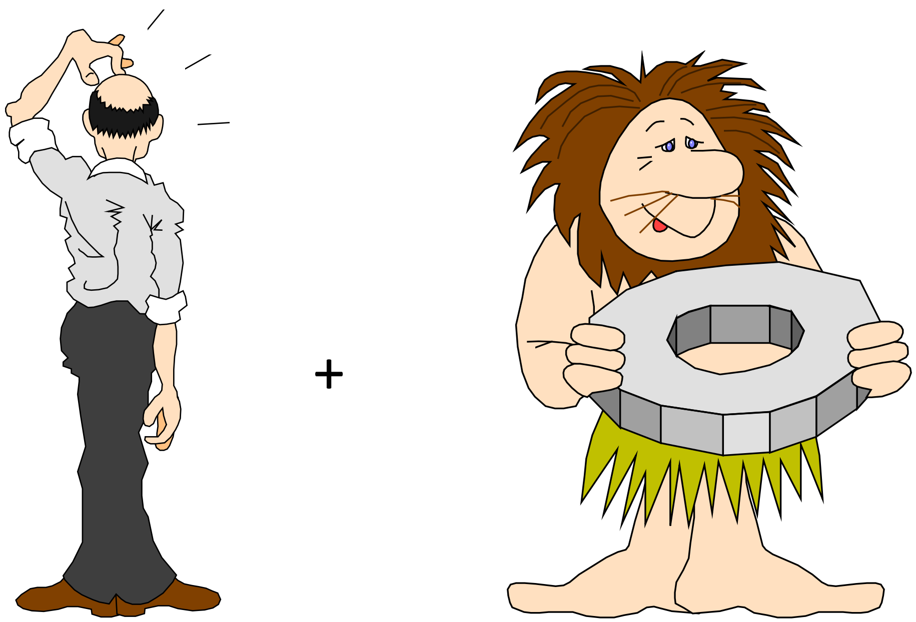
A cross section
ρ density, k constant

Identification

The model is obtained from input-output data collected from experiments, using some type of model fitting



Practical models



A practical model is always a mixture of knowledge and experimentation

Modelling / Simulation

Under which conditions can the model be used?

Model aims, precision,...

Hypothesis

↓

Modelling

Knowledge

↓

Parameterization

Experiments

↓

Coded for a certain computation

Coding

Simulation language

↓

Has been solved adequately?

Experimentation

All stages must be validated

Modelling methodology

Knowledge of the process

Aims , model type

Hypothesis

Model formulation

Parameter estimation

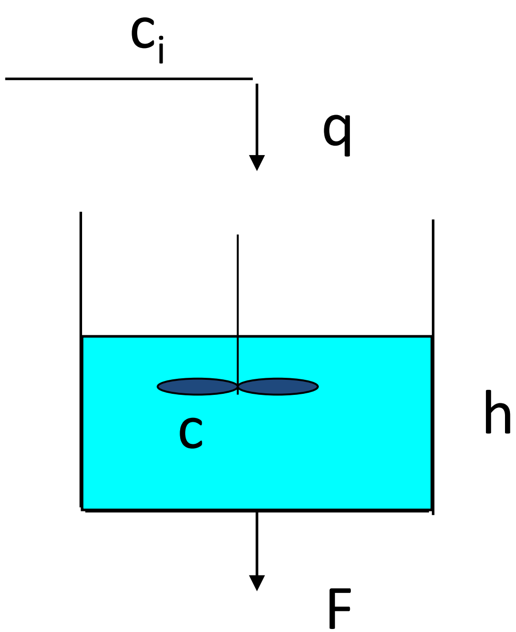
Model evaluation

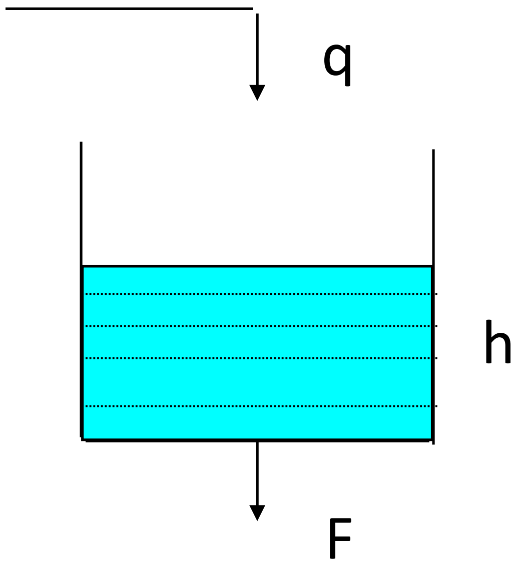
Explotation

Modelling requires specific techniques as well as human and material resources

time

Hypothesis





Perfect mixture

$$\frac{dVc}{dt} = qc_i - Fc$$

$$\frac{dV}{dt} = q - F \quad V = Ah$$

Plug flow

$$c(t) = c_i(t - \frac{h}{v}) = c_i(t - \frac{Ah}{Av}) = c_i(t - \frac{V}{F})$$

$$\frac{dV}{dt} = q - F \quad V = Ah$$

Continuous model types

- Lump models
- Distributed parameters
- No-linear
- Linear
- Time
- Frequency
- ….